PHILOSOPHICAL

TRANSACTIONS,

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXI.

PART I.

LONDON,

PRINTED BY W. BULMER AND CO. CLEVELAND-ROW, ST. JAMES'S;
AND SOLD BY G. AND W. NICOL, PALL-MALL, BOOKSELLERS TO HIS MAJESTY,
AND PRINTERS TO THE ROYAL SOCIETY.

MDCCCXI.

THE PRESIDENT and COUNCIL of the ROYAL SOCIETY adjudged the Gold and Silver Medals on COUNT RUMFORD's Donation to M. MALUS for his discoveries of certain new Properties of Reflected Light, published in the Second Volume of Mémoires d'Arcueil

ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the council-books and journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries, till the Forty-seventh Volume: the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March, 1752. And the grounds of their choice are, and will continue to

be, the importance and singularity of the subjects, or the advantageous manner of treating them, without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they receive them, are to be considered in no other light than as a matter of civility, in return for the respect shewn to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped, that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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PHILOSOPHICAL TRANSACTIONS.

I. *The Bakerian Lecture. On some of the Combinations of Oxymuriatic Gas and Oxygene, and on the chemical Relations of these Principles, to inflammable Bodies.* By Humphry Davy, *Esq. LL. D. Sec. R. S. F. R. S. E. M. R. I. A. and M. R. I.*

Read November 15, 1810.

1. *Introduction.*

In the last communication which I had the honour of presenting to the Royal Society, I stated a number of facts, which inclined me to believe, that the body improperly called in the modern nomenclature of chemistry, *oxymuriatic acid gas*, has not as yet been decompounded; but that it is a peculiar substance, elementary as far as our knowledge extends, and analogous in many of its properties to oxygene gas.

My objects in the present Lecture, are to detail a number of experiments which I have made for the purpose of illustrating more fully the nature, properties, and combinations of this substance, and its attractions for inflammable bodies, as compared with those of oxygene; and likewise to present

some general views and conclusions concerning the chemical powers of different species of matter, and the proportions in which they enter into union.

I have been almost constantly employed, since the last session of the Society, upon these researches, yet this time has not been sufficient to enable me to approach to any thing complete in the investigation. But on subjects, important both in their connexion with the higher departments of chemical philosophy, and with the œconomical applications of chemistry, I trust that even these imperfect labours will not be wholly unacceptable.

2. *On the Combinations of Oxymuriatic Gas and Oxygene, with the Metals from the fixed Alkalies.*

The intensity of the attraction of potassium for oxymuriatic gas, is shewn by its spontaneous inflammation in that substance, and by the vividness of the combustion. I satisfied myself, by various minute experiments, that no water is separated in this operation, and that the proportions of the compound are such that one grain of potassium absorbs about 1.1 cubical inch of oxymuriatic gas at the mean temperature and pressure, and that they form a neutral compound, which undergoes no change by fusion. I used, in the experiments from which these conclusions are drawn, a tray of platina for receiving the potassium; the metal was heated in an exhausted vessel, to decompose any water absorbed by the crust of potash, which forms upon the potassium during its exposure to the atmosphere, and the gas was freed from vapour by muriate of lime. Large masses of potassium cannot be made

to inflame, without heat in oxymuriatic gas. In all experiments in which I fused the potassium upon glass, the retorts broke in pieces in consequence of the violence of the combustion, and even in two instances when I used the tray of platina. If oxymuriatic gas be used, not freed from vapour, or if the potassium has been previously exposed to the air, a little moisture always separates during the process of combustion. When pure potassium, and pure oxymuriatic gas are used, the result, as I have stated, is a mere binary compound, the same as muriate of potash, that has undergone ignition.

The combustion of potassium and sodium in oxygene gas, is much less vivid than in oxymuriatic gas. From this phenomenon, and from some others, I was inclined to believe that the attraction of these metals for oxygene is feebler, than their attraction for oxymuriatic gas. I made several experiments, which proved that this is the fact; but before I enter upon a detail of them, it will be necessary to discuss more fully than I have yet attempted, the nature of the combinations of potassium and sodium with oxygene, and of potash and soda with water.

I have stated in the last Bakerian Lecture, that potassium and sodium, when burnt in oxygene gas, produce potash and soda in a state of extreme dryness, and very difficult of fusion. In the experiments from which these conclusions are drawn, as I mentioned, I used trays of platina, and finding that this metal was oxidated in the operation, I heated the retort strongly, to expel any oxygene the platina might have absorbed, and except in cases when this precaution was taken, I found the absorption of oxygene much greater than could be accounted for by the production of the alkalies. In all

cases in which I burnt potassium or sodium in common air, applying only a gentle heat, I found that the first products were substances extremely fusible, and of a reddish brown colour, which copiously effervesced in water, and which became dry alkali, by being strongly heated upon platina in the air, phenomena, which, at an early period of the enquiry, induced me to suppose that they were prot-oxides of potassium and sodium. Finding, in subsequent experiments, however, that they deflagrated with iron filings, and rapidly oxidated platina and silver, I suspended my opinion on the subject, intending to investigate their nature more fully.

Since that time, these oxides, as I find by a notice in the Moniteur for July 5th, 1810, have occupied the attention of M. M. GAY LUSSAC and THENARD, and these able chemists have discovered that they are peroxides of potassium and sodium, the one containing, according to them, three times as much oxygene as potash, and the other 1.5 times as much as soda

I have been able to confirm in a general way these interesting results, though I have not found any means of ascertaining accurately, the quantity of oxygene contained in these new oxides. When they are formed upon metallic substances, there is always a considerable oxidation of the metal, even though platina be employed. I have used a platina tray lined with muriate of potash, that had been fused; but in this case, though I am inclined to believe that some alkali was formed at the same time with the peroxides, yet I obtained an absorpton of 2.6 cubical inches, in a case when 2 grains of potassium were employed, and of 1.69 cubical inches, in a case when a grain of sodium was used, but in this last instance, the edge of the platina tray had been acted upon by the metal,

and was oxidated.* The mercury in the barometer in these experiments stood at 30.12 inches, and that in the thermometer at 62° FAHRENHEIT.

When these peroxides were formed upon muriate of potash, the colour of that from potassium was of a bright orange; that from sodium of a darker orange tint. They gave off oxygene, as M. M. GAY LUSSAC and THENARD state, by the action of water or acids. They were converted into alkali, as the French chemists have stated, by being heated with any metallic or inflammable matter. They thickened fixed oils, forming a compound that did not redden paper tinged with turmeric, without the addition of water.

When potassium is brought in contact with fused nitre, in tubes of pure glass, there is a slight scintillation only, and the nitre becomes of a red brown colour. In this operation, nitrogene is produced, and the oxide of potassium formed. I thought that by ascertaining the quantity of nitrogene evolved by the action of a given weight of potassium, and comparing this with the quantity of oxygene disengaged from the oxide by water, I might be able to determine its composition accurately. A grain of potassium acting in this way, I found produced only $\frac{15}{100}$ of nitrogene; and the red oxide by its action upon water, produced less than half a cubical inch of oxygene, so that it is probable that potash as well as its peroxide is formed in the operation.

* M. M. GAY LUSSAC and THENARD have stated in the paper above referred to, that common potash and barytes absorb oxygene when heated. It would seem that the action of the fixed alkalies, and of barytes on platina, depends on the production of the peroxides. I have little doubt but that these ingenious gentlemen will have anticipated this observation, in the detailed account of their experiments.

Sodium, when brought in contact with fused nitre, produced a violent deflagration. In two experiments in which I used a grain of the metal, the tube broke with the violence of the explosion. I succeeded in obtaining the solid results of the deflagration of $\frac{1}{3}$ a grain of sodium, but it appeared that no peroxide had formed, for the mass gave no oxygene by the action of water.

When potassium is burnt in a retort of pure glass, the result is partly potash and partly peroxide, and by a long continued red heat the peroxide is entirely decomposed.

A grain of potassium was gently heated in a small green glass retort containing oxygene; it burnt slowly, and with a feeble flame; a quantity of oxygene was absorbed equal to $\frac{90}{100}$ of a cubical inch; by heating the retort to dull redness, oxygene was expelled equal to $\frac{33}{100}$ of a cubical inch; the mercury in the thermometer in this experiment stood at 65° FAHRENHEIT, and that in the barometer at 30.1 inches.

In experiments on the electrical decomposition of potash and soda, when the VOLTAIC battery employed contains from 500 to 1000 series in full action; the metals burn at the moment of their production, and form the peroxides; and it is probable, from the observations of M. RITTER, that these bodies may be produced likewise in VOLTAIC operations on potash, at the positive surface.

In my early experiments on potassium and sodium, I regarded the fusible substances appearing at the negative surface, in the VOLTAIC circuit, as well as those produced by the exposure of the metals to heat and air, as prot-oxides, and as similar to the results obtained by heating the metals in contact with small quantities of alkali.

I have repeated these last operations, in which I conceived that prot-oxides were formed.

Potassium and sodium, when heated in glass tubes in contact with about half of their weight of potash and soda, that have been ignited, become first of a bright azure, then produce a considerable quantity of hydrogene, and at last form a gray coherent mass, not fusible at a dull red heat, and which gives hydrogene by the action of water.

Whether these are true prot-oxides, or merely mixtures of the alkaline metals with the alkalies, or with the alkalies and reduced silex from the glass, I shall not at present attempt to decide.

Potassium I find heated in a similar manner with fused potash, in a tube of platina, gives after having been ignited, a dark mass that effervesces with water; but even in this case, it may be said that the alloy of platina and potassium interferes, and that the substance is not a protoxide, but merely dry alkali mixed with this alloy.

As the pure alkalies were unknown, till the discovery of potassium and sodium,* and as their properties have never been described, it will perhaps be proper in this place to notice them briefly.

* Stahl approached nearly to the discovery of the pure alkalies. He cemented solid caustic potash with iron filings in a long continued heat, and states, that in this way an alkali "valde causticum" is produced. *Specim. Becb.* part ii. page 255. He procured caustic alkali also, by decomposing nitre by the metals. Id. p. 253.

I find that when nitre is decomposed in a crucible of platina, by a strong red heat, a yellow substance remains, which consists of potash and oxide of platina, apparently in chemical combination. The undecompounded potash which comes over in the process for procuring potassium by the gun-barrel, is of an olive colour, and affords oxide of iron during its solution in water. Pure potash will probably be found to have an affinity for many metallic oxides.

When potassium and sodium are burnt in oxygene gas upon platina, and heated to redness to decompose the peroxide of potassium, the alkalies are of a grayish green colour. They are harder than common potash or soda, and, as well as I could determine by an imperfect trial, of greater specific gravity. They require a strong red heat for their perfect fluidity, and evaporate slowly, by a still further increase of temperature. When small quantities of water are added to them, they heat violently, become white, and are converted into hydrats, and then are easily fusible and volatile.

When potassium or sodium is burnt on glass, freed from metallic oxides, and strongly heated, or when potash or soda is formed from the metals by the action of a minute quantity of water, their colour approaches to white; but in other sensible properties, they resemble the alkalies formed upon metallic substances; and are distinguished in a marked manner by their difficult fusibility from the potash and soda prepared by alcohol.

M. D'ARCET, and more distinctly M. BERTHOLLET, have concluded that the loss of weight of common fused potash and soda, during their combination with acids, depends upon the expulsion of water, which M. BERTHOLLET has rated at 18.9 per cent. for potash, and M. D'ARCET, at 27 or 28 for potash, and 28 or 29 for soda.*

I have stated in the last Bakerian Lecture, that my own results led me to conclude, that fused potash contained about 16 or 17 parts in the 100 of water, taking the potash formed, by adding oxygene to potassium as a standard.

The experiment from which I drew my conclusions, was made on the action of silex and potash fused together, and

* Annales de Chimie, tom. 68, page 190.

I regarded the loss of weight, as the indication of the quantity of moisture.

I am acquainted with no experiment on record, in which water has been actually collected from the ignited fixed alkalies, and this appeared necessary for the complete elucidation of the subject.

I heated together in a green glass retort, 40 grains of potash, (that had been ignited for several minutes), and 100 grains of boracic acid, which had been heated to whiteness for nearly an hour. The retort was carefully weighed, and connected with a small receiver, which was likewise weighed; the bulb of the retort was then gradually heated till it became of a cherry red; there was a violent effervescence in the retort, a fluid condensed in the neck, and passed into the receiver. When the process was completed, the whole of the retort was strongly heated; it was found to have lost 6½ grains, and the receiver had gained 5.8 grains. The fluid that it contained was water, holding in solution a minute quantity of boracic acid, and when evaporated, it did not leave an appreciable quantity of residuum.

A similar experiment made upon soda, heated to redness, but in which the water collected was not weighed, indicated 22.9 of water in 100 parts of soda.

It may be asked, whether part of the water evolved in these processes might not have been produced from the boracic acid, or formed in consequence of its agency; but the following experiments shew that this can not be the case in any sensible degree.

I heated 8 grains of potassium, with about 50 grains of boracic acid, to redness in a tube of platina, connected with a

glass tube, kept very cool; but I found that no moisture whatever was separated in the process. I mixed a few grains of potassium with red oxide of mercury, and ignited the mixture in contact with boracic acid, but no elastic product, except mercury, was evolved.

I made some potash by the combustion of potassium in a glass tube, and ignition of the peroxide, I added to it dry boracic acid, and heated the mixture to redness. Sub-borate of potash was formed, and there was not the slightest indications of the presence of moisture.*

It is evident from this chain of facts, that common potash

* These processes must not however be considered as shewing that boracic acid that has been heated to whiteness is entirely free from water; they merely prove that such an acid gives off no water by combination with pure potash at a *red heat*. I have found that boracic acid in perfect fusion, and that has been long exposed to the blast of a forge, and that has long ceased to effervesce, gives globules of hydrogene; when dry iron filings are made to act upon it. I added to 54 grains of boracic acid in complete fusion, in a crucible of platina, 75 grains of flint glass that had been previously heated to whiteness, and immediately reduced into powder in a hot iron mortar; by raising the heat so as to produce combination, a copious effervescence was produced; and after intense ignition for half an hour, the mixture was found to have lost three grains and a quarter.

The combinations of boracic acid with potash and soda, that have been heated to redness, I find lose weight when their temperature is raised to a much higher degree. Thus, in an experiment made in the laboratory of my friend JOHN GEORGE CHILDREN, ESQ. and in which Mr. CHILDREN was so kind as to co-operate, 71 grains of hydrat of potash, mixed with 96 of boracic acid that had been heated as strongly as possible in a blast furnace, lost by fusion together in a red heat 21 grains, but on raising the temperature to whiteness the loss increased to above 13 grains. 55.5 grains of hydrat of soda, mixed with 80 of boracic acid, examined at intervals in a process of this kind, continued to lose weight for half an hour, during which time they were frequently heated to whiteness; at the end of this period the whole loss was 14 grains, of which at least one grain and a half may be referred to the acid. 95 grains of soda, ignited to whiteness in a platina crucible, with 140 of dry flint glass, lost 21.2 grains; 80 grains of boracic glass were added to this mixture; a fresh effer-

and soda are hydrats, and the bodies formed by the combustion of the alkaline metals, are, as I have always stated, pure metallic oxides, (as far as our knowledge extends) free from water.*

vescence took place, and after intense ignition for a few minutes, there was an additional loss of weight of four grains and a half. The energy with which water adheres to certain bodies in other cases, is shewn by the experiments of M. BERTHOLLET, *Mem. d'Arcueil*, tom. ii. page 47. Indeed it is impossible to say that a neutral compound, or a fixed acid is ever entirely free from water; it is only the first proportions that are easily separated. If the proportions of water in common potash and soda were to be judged of from their loss of weight, in combining with boracic acid, it would appear to be from 19 to 20 per cent. in the first, and from 23 to 25 in the second.

* After the experiments detailed in my two last papers, it may perhaps appear unnecessary, at least to those enlightened British chemical philosophers who have closely followed the progress of science, to offer any new evidences to prove that potassium and sodium are not hydrurets of potash and soda, particularly as M. M. GAY LUSSAC and THENARD, the ingenious advocates of this notion have acknowledged, in the Moniteur to which I have before referred, that it is not tenable; but on a subject so intimately connected with the most refined departments of chemical philosophy, and with so many new objects of research, additional facts cannot be wholly devoid of use and application.

Mr. DALTON, in the second volume of the work which he entitles "*A New System of Chemical Philosophy*," of which he has had the goodness to send me a copy, has, I find in his first pages, adopted the idea that potash and soda are metallic oxides; but in the latter pages has considered them as simple bodies, and the metals formed from them as compounds of potash and soda with hydrogene. He has given no facts in favour of this change in his opinion: his principal argument is founded upon the process in which I first obtained potassium. Common potash is a hydrat: when oxygene is procured from this by VOLTAIC electricity at one surface, and potassium at the other surface; Mr. DALTON conceiving that this oxygene arises from the water, states that the hydrogene of the water must combine with the potash to form potassium. It is evident, that adopting such a plan of reasoning, lead or copper might be proved to be hydrurets of their oxides; for when these metals are revived from their aqueous acid solutions, oxygene is produced at the positive surface, and no hydrogene at the negative surface.

In my first experiments for producing potassium and sodium, I used a weak

I shall now resume the detail of the experiments that I have made, on the relative attractions of oxymuriatic gas and oxygene, for the metals of the fixed alkalies. I burnt a grain of potassium in oxygene gas, in a retort of green glass, furnished with a stop-cock, and heated the oxide formed, to redness, to convert it into potash: half a cubical inch of oxygene was absorbed. The retort was exhausted, and very pure oxymuriatic gas admitted. The

power, and in these instances procuring the metals in very small quantities only, I perceived no effervescence. When from five hundred to one thousand plates are used for producing potassium, there is a violent effervescence, and a production of hydrogene and sometimes of potassuretted hydrogene, connected with the formation of the metal.

Potassium, brought in contact with red hot hydrat of potash, disengages abundance of hydrogene, and the whole is converted into difficultly fusible potash.

327 grains of hydrat of potash that had been ignited, were made to act in a gun-barrel on 745 grains of iron turnings heated to whiteness. Some hydrogene was lost, and some hydrat of potash remained undecompounded, yet 225 cubical inches of inflammable gas were collected, and 50 grains of potassium, and a large quantity of an alloy of potassium and iron formed, so that it is scarcely possible to doubt that all the hydrogene produced from the decomposed hydrat of potash was liberated.

Mr. DALTON conceives that there is an analogy between potassium and sodium, and the compounds of hydrogene with sulphur, phosphorus, and arsenic; but I am at loss to trace any similarity between sulphureted hydrogene, which is a gaseous body, soluble in water, and having acid properties, and a highly inflammable solid metal which produces alkali by combustion. Potassium might as well be compared to carbonic acid. Mr. DALTON considers the volatility of potassium and sodium as favouring the idea of their containing hydrogene; but they are less volatile than antimony, arsenic, and tellurium, and much less volatile than mercury. He mentions their low specific gravity as a circumstance favourable to this idea. I have on a former occasion examined this argument, first brought forward by M. RITTER; but it may not be amiss to add, that if potassium is a compound of hydrogene and potash, hydrat of potash must contain an equal quantity of hydrogene, with the addition of a light gaseous element, oxygene, which might be expected to diminish rather than to increase the specific gravity of the compound. Mr. DALTON states, p. 488, that potassium forms dry hydrat of potash, by decomposing nitrous gas

colour of the potash instantly became white, and by a gentle heat, the whole was converted into muriate of potash: a cubical inch and $\frac{1}{8}$ of oxymuriatic gas were absorbed, and exactly half a cubical inch of oxygene generated. The barometer during this operation was at 30.3, the thermometer at 62 FAHRENHEIT. I made several experiments of the same kind, but this is the only one on which I can place entire dependence. When I attempted to use larger quantities of potassium, the retort usually broke during the cooling of the glass, and it was not possible to gain any accurate results in employing metallic trays. The potassium was spread into a thin plate, and of course was much oxidated before its admission into the retort, which rendered the absorption of oxygene a little less than it ought to have been. In the process it was heated in vacuo before the combustion, to decompose the water in the crust of potash; for in cases when this precaution was not taken, I found that hydrat of potash sublimed, and lined the upper part of the retort, and from this the oxymuriatic gas separated water as well as oxygene.

The phenomenon of the separation of water from hydrat of potash by oxymuriatic gas, was happily exemplified in an experiment in which I introduced oxymuriatic gas to the peroxide of potassium, formed in a large retort, and in which

and nitrous oxide; this is not the case: and he does not refer to experiment. I find by some very careful trials, that potassium attracts the oxygene and some of the nitrogene from these bodies, and forms a fusible compound which may be decomposed, giving off nitrogene and its excess of oxygene, by a red heat, and which becomes *potash*, and not dry hydrat of potash.

M. M. GAY LUSSAC and THENARD have convinced themselves that potassium and sodium are not hydrurets of potash and soda, by a method similar to that which I adopted and published some months before, namely, by producing neutral salts from them.

the potassium had been covered with a considerable crust of hydrat of potash. The upper part of the retort and its neck contained a white sublimate of hydrat, which had risen in combustion, and which was perfectly opaque. As soon as the gas was admitted, it instantly became transparent from the evolution of water; and on heating the glass in contact with the sublimate, its opacity was restored, and water driven off.

In various cases in which I heated dry potash, or mixtures of potash and the peroxide, in oxymuriatic gas, there was no separation of moisture, except when the gas contained aqueous vapour; and the oxygene evolved in the process, when the heat was strongly raised, exactly corresponded to that absorbed by the potassium.

When muriatic acid gas was introduced to potash formed from the combustion of potassium, water was instantly formed, and oxymuriate of potassium.* I have made no accurate experiment on the proportions of muriatic acid gas decomposed by potash, but I made a very minute investigation, of the nature of the mutual decomposition of this substance, and hydrat of potash.

Ten grains of hydrat of potash were heated to redness in a tray of platina, which was carefully weighed; it was introduced into a retort which was exhausted of air, and the retort was filled with muriatic acid gas. The hydrat of potash was heated by a spirit lamp; water instantly separated in great abundance, and muriate of potash formed. A strong heat was applied till the process was completed, when the tray was taken out and weighed; it had gained $2\frac{11}{16}$ grains. A minute quantity of liquid muriatic acid was added to the muriate, to

* i.e. Muriate of potash.

ensure a complete neutralization, and the tray heated to redness: there was no additional increase of weight.

In the few experiments which I have made on the action of sodium and soda on oxymuriatic gas, the phenomena appeared precisely analogous; but sodium, as might have been expected, absorbed nearly twice as much oxymuriatic gas as potassium.

When common salt that has been ignited, is heated with potassium, there is an immediate decomposition, and by giving the mixture a red heat, pure sodium is obtained; and this process affords an easy mode, and the one I have always lately adopted for procuring that metal. No hydrogene is disengaged in this operation, and two parts of potassium I find produces rather more than one of sodium.

From the series of proportions that I have communicated in my last paper, it is evident that 1 grain of potassium ought to absorb 1.08 cubical inches of oxymuriatic acid; and that the potash formed from one grain of potassium ought to decompose about 2.16 cubical inches of muriatic acid gas; and these estimations agree very nearly with the result of experiments.

The estimation of the composition of soda, as deduced from the experiments in the last Bakerian lecture, is 25.4 of oxygene to 74.6 of metal, and this would give the number representing the proportion in which sodium combines with bodies 22,* from which it is evident, that a grain of sodium

* Or if soda be considered as deutoxide, which seems probable from the experiments detailed page 4. 44; and on this supposion, the salts of soda must be conceived to contain double proportions of acid. On either datum the proportion of oxygene in water must be taken as 7.5, and that of hydrogene as 1, though other numbers might be found as divisors or multiples of those which would equally harmonise with the general doctrine of definite proportions. In my last communication to the Society, I have quoted Mr. DALTON as the original Author of the hypothesis,

ought to absorb nearly 2 cubical inches of oxymuriatic gas, and that the same quantity converted into soda, would decompose nearly four cubical inches of muriatic gas. Muriate of soda ought on this idea to contain one proportion of sodium, 22., and one of oxymuriatic gas 32.9; and this estimation is very near that which may be gained from Dr. MARCET's analysis of this substance. Hydrat of potash ought to consist of 1 proportion of potash, represented by 48., and one of water, represented by 8.5. This gives its composition as 15.1 of water, and 84.9 of potash. Hydrat of soda ought, according to theory, to contain 1 proportion of soda 29.5, and 1 of water 8.5, which will give in 100 parts 22.4 of water; and the experiments that I have detailed, conform as well as can be expected with these conclusions.

The proportions of potash and soda indicated, in different neutral combinations, by these estimations, will be found to agree very nearly with those derived from the most accurate analysis, particularly those of M. BERTHOLLET; or the differences are such as admit of an easy explanation.

I stated in my last communication, the probability that the oxygene in the hyperoxymuriate of potash was in triple combination with the metal and oxymuriatic gas; the new facts

that water consists of 1 particle of oxygene, and 1 of hydrogene; but I have since found that this opinion is advanced, in a work published in 1789. *A comparative View of the Phlogistic and Antiphlogistic Theories*, by WILLIAM HIGGINS. In this elaborate and ingenious performance, Mr. HIGGINS has developed many happy sketches of the manner in which (on the corpuscular hypothesis) the particles or molecules of bodies may be conceived to combine; and some of his views, though formed at this early period of investigation, appear to me to be more defensible, assuming his data, than any which have been since advanced; for instance, he considered nitrous gas as composed of two particles of oxygene, and one of nitrogene. Mr. HIGGINS had likewise drawn the just conclusion respecting the constitution of

respecting the peroxide confirm this idea. Potassium, perfectly saturated with oxygene, would probably contain six

sulphuretted hydrogene, from its electrical decomposition. As hydrogene is the substance which combines with other bodies in the smallest quantity, it is perhaps the most fitted to be represented by unity; and on this idea the proportions in ammonia will be 3 of hydrogene to 1 of nitrogene, and the number representing the smallest proportion in which nitrogene is known to combine will be 13.4. Mr. DALTON, *New System of Chemical Philosophy*, pages 323 and 436, has adopted 4.7 or 5.3, as the number representing the weight of the *atom* of nitrogene; and has quoted my experiments, *Researches, Chemical and Philosophical*, as authorising these numbers; but all the enquiries on nitric acid, nitrous gas, nitrous oxide, and on the decomposition of nitrat of ammonia stated in that work, conform much more nearly to the number 13.4.

According to Mr. DALTON, nitrat of ammonia contains one proportion of acid and one of alkali, and nitrate of potash two proportions of acid and one of alkali; but it is easy to see that the reverse must be the case. Nitrate of ammonia is known to be an acid salt; and nitrate of potash a neutral salt; which harmonizes with the views above stated. Mr. DALTON estimates the quantity of water in nitric acid of specific gravity 1.54, at 27.5 per cent.; and this, according to him, is a stronger acid than he obtained by decomposing fused nitre by sulphuric acid, which contained only 19 per cent. of water, and one quantity of sulphuric acid, according to him, will produce from nitre, more than an equal weight of nitric acid, and he supposes no water in nitre; so that his conclusion as to the quantity of water in liquid nitric acid on his own data must be incorrect. I find water in fused nitre, by decomposing it by boracic acid.

I shall enter no further at present into an examination of the opinions, results, and conclusions of my learned friend; I am however obliged to dissent from most of them, and to protest against the interpretations that he has been pleased to make of my experiments; and I trust to his judgment and candour for a correction of his views.

It is impossible not to admire the ingenuity and talent with which Mr. DALTON has arranged, combined, weighed, measured, and figured his atoms; but it is not, I conceive, on any speculations upon the ultimate particles of matter, that the true theory of definite proportions must ultimately rest. It has a surer basis in the mutual decomposition of the neutral salts, observed by RICHTER and GUYTON DE MORVEAU, in the mutual decompositions of the compounds of hydrogene and nitrogene, of nitrogene and oxygene, of water and the oxymuriatic compounds; in the multiples of oxygene in the nitrous compounds; and those of acids in salts, observed by Drs. WOLLASTON

proportions; for, according to Mr. CHENEVIX's analysis, which is confirmed by one made in the Laboratory of the Royal Institution, by Mr. E. DAVY, hyperoxymuriate of potash must consist of 40.5 potassium, 32.9 oxymuriatic gas, and 45 of oxygene.

I have mentioned, that by strongly heating the peroxide of potassium in oxymuriatic acid, all the oxygene is expelled, and a mere combination of oxymuriatic gas and potassium formed. I thought it possible, that at a low temperature, a combination might be effected, and I have reason to believe that this is the case. I made a peroxide of potassium, by heating potassium with about twice the quantity of nitre, and admitted oxymuriatic gas which was absorbed: some oxygene was expelled on the fusion of the peroxide, but a salt remained, which gave oxymuriatic gas, as well as muriatic acid, by the action of sulphuric acid.

It seems evident, that in the formation of the hyperoxymuriate of potash, one quantity of potash is decomposed by the attraction of oxymuriatic gas to form muriate of potash; but the oxygene, instead of being set free in the nascent state, enters into combination with another portion of potash, to form a peroxide, and with oxymuriatic gas.

The proportions required for these changes may be easily deduced from the data which have been stated in the preceding pages. 5 proportions of potash, equal to 240 grains, must be decomposed to form with an equal number of proportions

and THOMSON; and above all, in the decompositions by the VOLTAIC apparatus. Where oxygene and hydrogene, oxygene and inflammable bodies, acids and alkalies, &c. must separate in uniform ratios.

of oxymuriatic gas equal to 164.5 grains, 5 proportions of muriate of potash equal to 367 grains; and 5 of oxygene equal to 37,5 grains, combined with one of potash, equal to 48, must unite in triple union with one of oxymuriatic gas equal to 32.9, to form one proportion, equal to 118.4 grains of hyperoxymuriate of potash.

3. *On the Combinations of the Metals of the Earths, with Oxygene and Oxymuriatic Gas.*

The muriates of baryta, lime, and strontia, after being a long time in a white heat, are not decomposable by any simple attractions: thus, they are not altered by dry boracic acid, though, when water is added to them, they readily afford muriatic acid and their peculiar earths.

From this circumstance, I was induced to believe that these three compounds consist merely of the peculiar metallic bases, which I have named barium, strontium, and calcium, and oxymuriatic gas; and such experiments as I have been able to make, confirm the conclusion.

When baryta, strontia, or lime, is heated in oxymuriatic gas to redness, a body precisely the same as a dry muriate is formed, and oxygene is expelled from the earth. I have never been able to effect so complete a decomposition of these earths by oxymuriatic gas, as to ascertain the quantity of oxygene produced from a given quantity of earth. But in three experiments made with great care I found that one of oxygene was evolved for every two in volume of oxymuriatic gas absorbed.

I have not yet tried the experiment of acting upon oxymuriatic gas by the bases of the alkaline earths; but I have

not the least doubt that these bodies would combine directly with that substance, and form dry muriates.

In the last experiments that I made on the metallization of the earths by amalgamation, I paid particular attention to the state of the products formed, by exposing the residuum of amalgams to the air. I found that baryta formed in this way was not fusible at an intense white heat, and that strontia and lime so formed gave off no water when ignited. Baryta made from chrystals of the earth, as M. BERTHOLLET has shewn, is a fusible hydrat, and I found that this earth gave moisture when decomposed by oxymuriatic gas; and the lime, in hydrat of lime, was much more rapidly decomposed by oxymuriatic gas than quicklime, its oxygene being rapidly expelled with the water.

Some dry quicklime was heated in a retort, filled with muriatic acid gas; water was instantly formed in great abundance, and it can hardly be doubted, that this arose from the hydrogene of the acid combining with the oxygene of the lime.

As potassium so readily decomposes common salt, I thought it might possibly decompose muriate of lime, and thus afford easy means of procuring calcium. The rapidity with which muriate of lime absorbs water, and the difficulty of freeing it even by a white heat from the last portions, rendered the circumstances of the experiments unfavourable. I found, however, that by heating potassium strongly, in contact with the salt, in a retort of difficultly fusible glass, I obtained a dark coloured matter, diffused through a vitreous mass, which effervesced strongly with water. The potassium had all disappeared, and the retort had received a heat at which potassium

entirely volatilizes. I had similar results with muriate of strontia, and (though less distinct, more potassium distilling off unaltered) with muriate of baryta. Either the bases of the earths were wholly or partially deprived of oxymuriatic gas in these processes, or the potassium had entered into triple combination with the muriates. I hope on a future occasion to be able to decide this point.

Combinations of muriatic acid gas with magnesia, alumine and silex, are all decomposed by heat, the acid being driven off, and the earth remaining free. I conjectured from this circumstance, that oxymuriatic gas would not expel oxygene from these earths, and the suspicion was confirmed by experiments. I heated magnesia, alumine, and silex to redness in oxymuriatic gas, but no change took place.

M. M. Gay Lussac and Thenard have shewn that baryta is capable of absorbing oxygene; and it seems likely, (as according to Mr. Chenevix's experiments, most of the earths are capable of becoming hyperoxymuriates) that peroxides of their bases must exist.

I endeavoured to combine lime with more oxygene, by heating it in hyperoxymuriate of potash, but without success, at least after this process it gave off no oxygene in combining with water. The salt, called oxymuriate of lime, made for the use of the bleachers, I found gave off oxygene by heat, and formed muriate of lime.

From the proportions which I have given in the last Bakerian lecture, but which were calculated from the analyses of sulphates, it follows that if the muriates of baryta, strontia, and lime, be regarded as containing one proportion

of oxymuriatic gas, and one of metal, then they would consist of 71* barium, 46 strontium, and 21 calcium, to 32.9 of oxymuriatic gas.

To determine how far these numbers are accurate, 50 grains of each of these muriates that had been heated to whiteness were decomposed by nitrate of silver, the precipitate was collected, washed, heated, and weighed.

The muriate of baryta, treated in this way, afforded 68 grains of horn-silver.

The muriate of strontia 85 grains.

The muriate of lime 125 grains.

From experiments to be detailed in the next section, it appears that horn-silver consists of 12 of silver to 3.9 of oxymuriatic gas, and consequently that barium should be represented by 65.1, strontium by 46.1, and calcium by 20.8.

4. *On the Combinations of the Common Metals, with Oxygene and Oxymuriatic Gas.*

In the limits which it is usual to adopt in this lecture, it will not be possible for me to give more than an outline of the numerous experiments that I have made on the combinations of oxymuriatic gas with metals; I must confine myself to a general statement of the mode of operating, and the results. I used in all cases small retorts of green glass, containing from 3 to 6 cubical inches, furnished with stopcocks. The metallic substances were introduced, the retort exhausted and filled with the gas to be acted upon, heat was applied by

* If Mr. JAMES THOMPSON's analysis of sulphate of barytes be made the basis of calculation, sulphuric acid being estimated as 36, then the number representing barium will be about 65.5,

means of a spirit lamp, and after cooling, the results were examined, and the residual gas analysed.

All the metals that I tried, except silver, lead, nickel, cobalt, and gold, when heated, burnt in the oxymuriatic gas, and the volatile metals with flame. Arsenic, antimony, tellurium and zinc with a white flame, mercury with a red flame. Tin became ignited to whiteness, and iron and copper to redness; tungsten and manganese to dull redness; platina was scarcely acted upon at the heat of fusion of the glass.

The product from arsenic was butter of arsenic; a dense, limpid, highly volatile fluid, a non-conductor of electricity, and of high specific gravity, and which when decomposed by water, gave oxide of arsenic and muriatic acid. That from antimony, was butter of antimony, an easily fusible and volatile solid, of the colour of horn-silver, of great density, crystallizing on cooling in hexahedral plates, and giving, by its decomposition by water, white oxide.

The product from tellurium, in its sensible qualities, resembled that from antimony, and gave when acted on by water white oxide.

The product from mercury was corrosive sublimate. That from zinc was similar in colour to that from antimony, but was much less volatile.

The combination of oxymuriatic gas and iron, was of a bright brown; but having a lustre approaching to the metallic, and was iridescent like the Elba iron ore. It volatilized at a moderate heat, filling the vessel with beautiful minute crystals of extraordinary splendour, and collecting in brilliant plates, the form of which I could not determine. When acted on by water, it gave red muriate of iron.

Copper formed a bright red brown substance, fusible at a heat below redness, and becoming crystalline and semi-transparent on cooling, and which gave a green fluid, and a green precipitate by the action of water.*

The substance from manganese was not volatile at a dull red heat; it was of a deep brown colour, and by the action of water became of a brighter brown: a muriate of manganese, which did not redden litmus remained in solution; and an insoluble matter remained of a chocolate colour.†

Tungsten afforded a deep orange sublimate, which, when decomposed by water, afforded muriatic acid, and the yellow oxide of tungster.

Tin afforded LIBAVIUS's liquor, which gave a muriate by the action of water containing the oxide of tin, at the maximum of oxidation.

Silver and lead produced horn-silver and horn-lead, and bismuth, butter of bismuth. The absorption of oxymuriatic gas was in the following proportions for two grains of each of the metals: for arsenic 3.6 cubical inches, for antimony

* It is worth enquiry, whether the precipitate from oxymuriate of copper by water is not a hydrated submuriate, analogous in its composition to the crystalized muriate of Peru. This last I find affords muriatic acid and water by heat.

The *resin of copper* discovered by BOYLE, formed by heating copper with corrosive sublimate, probably contains only a proportion of oxymuriatic gas, whilst that above referred to must contain 2.

† When muriate of manganese is made by solution of its oxide in muriatic acid, a neutral combination is obtained, but this is decomposed by heat; muriatic gas flies off, and brown oxide of manganese remains. In this respect manganese appears as a link between the ancient metals and the newly discovered ones. Its muriate is decomposed like that of magnesia; and its oxide is the only one amongst those long known, as far as my experiments have gone, which neutralizes the acid energy of muriatic acid gas, so as to prevent it in solution from affecting vegetable blues.

3.1, for tellurium 2.4, for mercury 1.05,* for zinc 3.2, for iron 5.8, for tin 4, for bismuth 1.5, for copper 3.4, for lead 9, for silver, the absorption of volume was $\frac{2}{10}$, and the increase of weight of the silver was equivalent to $\frac{6}{10}$ of a grain.†

In acting upon metallic oxides by oxymuriatic gas, I found that those of lead, silver, tin, copper, antimony, bismuth, and tellurium, were decomposed in a heat below redness, but the oxides of the volatile metals, more readily than those of the fixed ones. The oxides of cobalt and nickel were scarcely acted upon at a dull red heat. The red oxide of iron was not affected at a strong red heat, whilst the black oxide was rapidly decomposed at a much lower temperature; arsenical acid underwent no change at the greatest heat that could be given it in the glass retort, whilst the white oxide readily decomposed.

In cases where oxygene was given off, it was found exactly the same in quantity as that which had been absorbed by the metal. Thus 2 grains of red oxide of mercury absorbed $\frac{2}{10}$ of a cubical inch of oxymuriatic gas, nd afforded 0.45 of oxygene.‡ Two grains of dark olive oxide from calomel decom-

* The gas in these experiments was not freed from aqueous vapour, and as stopcocks of brass were used, a little gas might have been absorbed by the surface of this metal, so that the processes offer only approximations to the composition of the oxymuriates. The processes on lead, tellurium, iron, antimony, copper, tin, mercury, and arsenic, were carried on in three successive days, during which the height of the mercury in the barometer varied from 30.26 inches to 30.15, and the height of that in the thermometer from 63.5 to 61 FAHRENHEIT.

The experiment on silver was made at the temperature of 52 FAHRENHEIT, and under a pressure equal to that of 29.9 inches.

† This agrees nearly with another experiment made by my brother, Mr. JOHN DAVY, in which 12 grains of silver increased to 15.9 during their conversion into horn-silver.

‡ I have made two analyses of corrosive sublimate and calomel, with considerable

posed by potash, absorbed about $\frac{22}{100}$ of oxymuriatic gas, and afforded $\frac{11}{100}$ of oxygene, and corrosive sublimate was produced in both cases.

In the decomposition of the white oxide of zinc, oxygene was expelled exactly equal to half the volume of the oxymuriatic acid absorbed. In the case of the decomposition of the black oxide of iron, and the white oxide of arsenic, the changes that occurred were of a very beautiful kind; no oxygene was given off in either case, but butter of arsenic, and arsenical acid formed in one instance, and the ferruginous sublimate, and red oxide of iron in the other.

care. I decomposed 100 grains of corrosive sublimate, by 90 grains of hydrat of potash. This afforded 79.5 grains of orange coloured oxide of mercury, 40 grains of which afforded 9.15 cubical inches of oxygene gas; the muriate of silver formed from the 100 grains was 102.5.

100 grains of calomel, decomposed by 90 grains of potash, afforded 82 grains of olive coloured oxide of mercury, of which 40 grains gave by decomposition by heat 4.8 cubical inches of oxygene. The quantity of horn-silver formed from the 100 grains was 58.75 grains.

In the second analysis, the quantity of oxide obtained from corrosive sublimate was 78.7; the quantity of muriate of silver formed was 103.4; the oxide produced from calomel weighed 83 grains; the horn-silver formed was 57½ grains. I am inclined to put most confidence in the last analyses; but the tenor of both is to shew that the quantity of oxymuriatic gas in corrosive sublimate, is exactly double that in calomel, and that the orange oxide contains twice as much oxygene as the black, the mercury being considered as the same in all. The olive colour of the oxide formed from calomel, is owing to a slight admixture of orange oxide, formed by the oxygene of the water used in precipitation; the tint I find is almost black, when a boiling solution of potash is used; and trituration, with a little orange oxide brings the tint to olive. It has been stated, that the olive oxide thrown down from calomel by potash is a submuriate; but I have never been able to find a vestige of muriatic acid in it when well washed. It is not easy to obtain perfect precision in analyses of the oxides of mercury; water adheres to the oxides, which cannot be entirely driven off without the expulsion of some oxygene. In all my experiments, though the oxides had been heated to a temperature above 212, a little dew collected in the neck of the retort, so that the 40 grains must have been over-rated.

Two grains of white oxide of arsenic absorbed 0.8 of oxymuriatic gas.*

I doubt not that the same phenomena will be found to occur in other instances, in which the metal has comparatively a slight attraction only for oxymuriatic gas, and when it is susceptible of different degrees of oxydation, and in which the peroxide is used.

The only instance in which I tried to decompose a common metallic oxide, by muriatic acid, was in that of the fawn coloured oxide of tin; water rapidly separated, and LIBAVIUS's liquor was formed.

From the proportions which may be gained in considering the volumes of oxymuriatic gas absorbed by the different metals, in their relations to the quantity of oxygene which would be required to convert them into oxides, it would appear, that in the experiments to which I have referred, either one, two, or three proportions of oxymuriatic gas combine with one of metal, and consequently, from the composition of the muriates, it will be easy to obtain the numbers representing the proportions in which these metals may be conceived to enter into other compounds.†

* A singular instance of the tendency of the oxide of arsenic to become arsenical acid, occurs in its action on fused hydrat of potash, the water in the hydrat is rapidly decomposed, and arseniuretted hydrogene evolved, and arseniate of potash formed.

† From the experiments detailed in the note in the opposite page, it would appear that the number representing the proportion in which mercury combines must be about 200. That of silver, as would appear from the results, page 25, about 100. The numbers of other metals may be learnt from the data in the same page, but from what has been stated, these data cannot be considered as very correct.

5. *General Conclusions and Observations, illustrated by Experiments.*

All the conclusions which I ventured to draw in my last communication to the Society, will, I trust, be found to be confirmed by the whole series of these new enquiries.

Oxymuriatic gas combines with inflammable bodies, to form simple binary compounds; and in these cases, when it acts upon oxides, it either produces the expulsion of their oxygene, or causes it to enter into new combinations.

If it be said that the oxygene arises from the decomposition of the oxymuriatic gas, and not from the oxides, it may be asked, why it is always the quantity contained in the oxide; and why in some cases, as those of the peroxides of potassium and sodium, it bears no relation to the quantity of gas?

If there existed any acid matter in oxymuriatic gas, combined with oxygene, it ought to be exhibited in the fluid compound of one proportion of phosphorus, and two of oxymuriatic gas; for this, on such an assumption, should consist of muriatic acid (on the old hypothesis, free from water) and phosphorous acid; but this substance has no effect on litmus paper, and does not act under common circumstances, on fixed alkaline bases, such as dry lime or magnesia. Oxymuriatic gas, like oxygene, must be combined in large quantity with peculiar inflammable matter, to form acid matter. In its union with hydrogene, it instantly reddens the driest litmus paper, though a gaseous body. Contrary to acids, it expels oxygene from protoxides, and combines with peroxides.

When potassium is burnt in oxymuriatic gas, a dry compound is obtained. If potassium combined with oxygene is

employed, the whole of the oxgene is expelled, and the same compound formed. It is contrary to sound logic to say, that this exact quantity of oxygene is given off from a body not known to be compound, when we are certain of its existence in another; and all the cases are parallel.

An argument in favour of the existence of oxygene in oxymuriatic gas, may be derived by some persons from the circumstances of its formation, by the action of muriatic acid on peroxides, or on hyperoxymuriate of potash; but a minute investigation of the subject will, I doubt not, shew that the phænomena of this action are entirely consistent with the views I have brought forward. By heating muriatic acid gas in contact with dry peroxide of manganese, water I found was rapidly formed, and oxymuriatic gas produced, and the peroxide rendered brown. Now as muriatic acid gas is known to consist of oxymuriatic gas and hydrogene, there is no simple explanation of the result, except by saying that the hydrogene of the muriatic acid, combined with oxygene from the peroxide to produce water.

Scheele explained the bleaching powers of the oxymuriatic gas, by supposing that it destroyed colours by combining with phlogiston. Berthollet considered it as acting by supplying oxygene. I have made an experiment, which seems to prove that the pure gas is incapable of altering vegetable colours, and that its operation in bleaching depends entirely upon its property of decomposing water, and liberating its oxygene.

I filled a glass globe containing dry powdered muriate of lime, with oxymuriatic gas. I introduced some dry paper tinged with litmus that had been just heated, into another globe containing dry muriate of lime; after some time this globe

was exhausted, and then connected with the globe containing the oxymuriatic gas, and by an appropriate set of stopcocks, the paper was exposed to the action of the gas. No change of colour took place, and after two days there was scarcely a perceptible alteration.

Some similar paper dried, introduced into gas that had not been exposed to muriate of lime, was instantly rendered white.*

Paper that had not been previously dried, brought in contact with dried gas, underwent the same change, but more slowly.

The hyperoxymuriates seem to owe their bleaching powers entirely to their loosely combined oxygene; there is a strong tendency in the metal of those in common use, to form simple combinations with oxymuriatic gas, and the oxygene is easily expelled or attracted from them.

It is generally stated in chemical books, that oxymuriatic gas is capable of being condensed and crystallized at a low temperature; I have found by several experiments that this is not the case. The solution of oxymuriatic gas in water freezes more readily than pure water, but the pure gas dried by muriate of lime undergoes no change whatever, at a temperature of 40 below 0° of FAHRENHEIT. The mistake seems to have arisen from the exposure of the gas to cold in bottles containing moisture.

I attempted to decompose boracic and phosphoric acids by oxymuriatic gas, but without success; from which it seems probable, that the attractions of boracium and phosphorus for

* The last experiments were made in the laboratory of the Dublin Society, most of the preceding ones in the laboratory of the Royal Institution; and I have been permitted to refer to them by the Managers of that useful public establishment.

oxygene are stronger than for oxymuriatic gas. And from the experiments I have already detailed, iron and arsenic are analogous in this respect, and probably some other metals.

Potassium, sodium, calcium, strontium, barium, zinc, mercury, tin, lead, and probably silver, antimony, and gold seem to have a stronger attraction for oxymuriatic gas than for oxygene.

I have as yet been able to make very few experiments on the combinations of the oxymuriatic compounds with each other, or with oxides. The liquor from arsenic, and that from tin, mix, producing an increase of temperature; and the phosphuretted, and the sulphuretted liquors unite with each other, and with the liquor of Libavius, but without any remarkable phenomena.

I heated lime gently in a green glass tube, and passed the phosphoric sublimate, the saturated oxymuriate of phosphorus through it, in vapour; there was a violent action with the production of heat and light, and a gray fused mass was formed, which afforded by the action of water, muriate and phosphate of lime.

I introduced some vapour from the heated phosphoric sublimate, into an exhausted retort containing dry paper tinged with litmus: the colour slowly changed to pale red. This fact seems in favour of the idea that the substance is an acid; but as some minute quantity of aqueous vapour might have been present in the receiver, the experiment cannot be regarded as decisive: the strength of its attraction for ammonia, is perhaps likewise in favour of this opinion. All the oxymuriates that I have tried, indeed form triple compounds with this alkali; but phosphorus is expelled by a gentle

heat from the other compounds of oxymuriatic gas and phosphorus with ammonia, and the substance remaining in combination is the phosphoric sublimate.

6. *Some Reflections on the Nomenclature of the Oxymuriatic Compounds.*

To call a body which is not known to contain oxygene, and which cannot contain muriatic acid, oxymuriatic acid, is contrary to the principles of that nomenclature in which it is adopted; and an alteration of it seems necessary to assist the progress of discussion, and to diffuse just ideas on the subject. If the great discoverer of this substance had signified it by any simple name, it would have been proper to have recurred to it; but, dephlogisticated marine acid is a term which can hardly be adopted in the present advanced æra of the science.

After consulting some of the most eminent chemical philosophers in this country, it has been judged most proper to suggest a name founded upon one of its obvious and characteristic properties—its colour, and to call it *Chlorine*, or *Chloric* gas.*

Should it hereafter be discovered to be compound, and even to contain oxygene, this name can imply no error, and cannot necessarily require a change.

Most of the salts which have been called muriates, are not known to contain any muriatic acid, or any oxygene. Thus LIBAVIUS's liquor, though converted into a muriate by water, contains only tin and oxymuriatic gas, and horn-silver seems incapable of being converted into a true muriate.

I venture to propose for the compounds of oxymuriatic

* From χλωρος.

gas and inflammable matter, the name of their bases, with the termination *ane*. Thus argentane may signify horn-silver; stannane, LIBAVIUS's liquor; antimonane, butter of antimony; sulphurane, DR. THOMSON's sulphuretted liquor; and so on for the rest.

In cases when the proportion is one quantity of oxymuriatic gas, and one of inflammable matter, this nomenclature will be competent to express the class to which the body belongs, and its constitution. In cases when two or more proportions of inflammable matter, combine with one of gas; or two or more of gas, with one of inflammable matter, it may be convenient to signify the proportions by affixing vowels before the name, when the inflammable matter predominates, and after the name, when the gas is in excess; and in the order of the alphabet, *a* signifying two, *e* three, *i* four, and so on.

The name muriatic acid, as applied to the compound of hydrogene and oxymuriatic gas, there seems to be no reason for altering. And the compounds of this body with oxides should be characterized in the usual manner, and as the other neutral salts.

Thus muriate of ammonia and muriate of magnesia, are perfectly correct expressions.

I shall not dwell any longer at present upon this subject.—What I have advanced, I advance merely as suggestion, and principally, for the purpose of calling the attention of philosophers to it.* As chemistry improves, many other alterations

* It may be conceived that a name may be found for the oxymuriatic gas in some modification of its present appellation which may harmonize with the new views, and which may yet signify its relation to the muriatic acid, such as demuriatic gas, or oxymuric gas; but in this case it would be necessary to call the muriatic acid, hydrogenated muriatic acid, or hydromuriatic acid; and the salts which contain it

will be necessary; and it is to be hoped that whenever they take

hydrogenated muriates or hydromuriates; and on such a plan, the compounds of oxymuriatic gas must be called demuriates or oxymuriates, which I conceive would create more complexity and difficulty in unfolding just ideas on this department of chemical knowledge than the methods which I have ventured to propose. It may however be right, considering the infant state of the investigation, to suspend, for a a time, the adoption of any new terms for these compounds. It is possible that oxymuriatic gas may be compound, and that this body and oxygene may contain some common principle; but at present we have no more right to say that oxymuriatic gas contains oxygene than to say that tin contains hydrogene; and names should express things and not opinions; and till a body is decompounded, it should be considered as simple.

In the last number of Mr. NICHOLSON's Journal, which appeared February 1st, whilst this sheet was correcting for the press, I have seen an ingenious paper, by Mr. MURRAY, of Edinburgh, in which he has attempted to shew, that oxymuriatic gas contains oxygene. His methods are, by detonating oxymuriatic gas in excess, with a mixture of hydrogene, and gaseous oxide of carbone, when he *supposes* carbonic acid is formed; and by mixing oxymuriatic gas in excess, with sulphuretted hydrogene, when he *supposes* sulphuric acid, or sulphureous acid is formed. In some experiments, in which my brother, Mr. JOHN DAVY, was so good as to co-operate, made over boiled mercury, we found, that 7 parts of hydrogene, 8 parts of gaseous oxide of carbone, and 20 parts of oxymuriatic gas, exploded by the electric spark, diminished to about 30 measures; and calomel was formed on the sides of the tube. On adding dry ammonia in excess, and exposing the remainder to water, a gas remained, which equalled more than 9 measures, and which was gaseous oxide of carbone, with no more impurity than might be expected from the air in the gasses, and the nitrogene expelled from the ammonia; so that the oxygene in Mr. MURRAY's carbonic acid, it seems, was obtained from *water*, or from the carbonic oxide. Sulphuretted hydrogene, added over dry mercury, to oxymuriatic gas in excess, inflamed in two or three experiments; muriatic acid gas containing the vapour of oxymuriate of sulphur, was formed, which, when neutralized by ammonia, gave muriate of ammonia, and a combination of ammonia, and oxymuriate of sulphur.

When a mixture of oxymuriatic gas in excess, and sulphuretted hydrogene, was suffered to pass into the atmosphere, the smell was that of oxymuriate of sulphur; there was not the slightest indication of the presence of any sulphuric or sulphureous acid. If Mr. MURRAY had used ammonia, instead of water, for analyzing his results, I do not think he would have concluded, that oxymuriatic gas is capable of decomposition by such methods.

I shall not, at present, enter upon a detail of other experiments which I have made

place, they will be made independent of all speculative views, and that new names will be derived from some simple and invariable property, and that mere arbitrary designations will be employed, to signify the class to which compounds or simple bodies belong.

on this subject, in co-operation with my brother, as it is his intention to refer to them, in an answer to Mr. MURRAY's paper.

I shall conclude, by saying, that this ingenious chemist, has mistaken my views, i supposing them hypothetical; I merely state what I have seen, and what I have found There *may* be oxygene in oxymuriatic gas; but I can find none. I repeated Mr. MURRAY's experiments with great interest; and their results, when *water* is excluded, entirely confirm all my ideas on the subject, and afford no support to the hypothetical ideas, which he has laboured so zealously to defend.

II. *The Croonian Lecture, on some Physiological Researches, respecting the Influence of the Brain on the Action of the Heart, and on the Generation of animal Heat.* By *Mr.* B. C. Brodie, *F.R.S.*

Read December 20, 1810.

HAVING had the honour of being appointed, by the President of the Royal Society, to give the Croonian Lecture, I trust that the following facts and observations will be considered as tending sufficiently to promote the objects, for which the Lecture was instituted. They appear to throw some light on the mode, in which the influence of the brain is necessary to the continuance of the action of the heart; and on the effect, which the changes produced on the blood in respiration have on the heat of the animal body.

In making experiments on animals to ascertain how far the influence of the brain is necessary to the action of the heart, I found that when an animal was pithed by dividing the spinal marrow in the upper part of the neck, respiration was immediately destroyed, but the heart still continued to contract circulating dark-coloured blood, and that in some instances from ten to fifteen minutes elapsed before its action had entirely ceased. I further found that when the head was removed, the divided blood vessels being secured by a ligature, the circulation still continued, apparently unaffected by the entire separation of the brain. These experiments confirmed the

observations of Mr. CRUIKSHANK * and M. BICHAT,† that the brain is not directly necessary to the action of the heart, and that when the functions of the brain are destroyed, the circulation ceases only in consequence of the suspension of respiration. This led me to conclude, that, if respiration was produced artificially, the heart would continue to contract for a still longer period of time after the removal of the brain. The truth of this conclusion was ascertained by the following experiment.

Experiment 1.

I divided the spinal marrow of a rabbit in the space between the occiput and atlas, and having made an opening into the trachea, fitted into it a tube of elastic gum, to which was connected a small pair of bellows, so constructed that the lungs might be inflated, and then allowed to empty themselves. By repeating this process once in five seconds, the lungs being each time fully inflated with fresh atmospheric air, an artificial respiration was kept up. I then secured the blood-vessels in the neck, and removed the head, by cutting through the soft parts above the ligature, and separating the occiput from the atlas. The heart continued to contract, apparently with as much strength and frequency as in a living animal. I examined the blood in the different sets of vessels, and found it dark-coloured in the venæ cavæ and pulmonary artery, and of the usual florid red colour in the pulmonary veins and aorta. At the end of twenty-five minutes from the time of the spinal marrow being divided, the action of the heart became fainter, and the experiment was put an end to.

* Philosophical Transactions 1795.

† Récherches Physiologiques sur la Vie et la Mort.

With a view to promote the enquiry instituted by the Society for promoting the knowledge of animal chemistry respecting the influence of the nerves on the secretions,* I endeavoured to ascertain whether they continued after the influence of the brain was removed. In the commencement of the experiment I emptied the bladder of its contents by pressure; at the end of the experiment the bladder continued empty.

This experiment led me to conclude, that the action of the heart might be made to continue after the brain was removed, by means of artificial respiration, but that under these circumstances the secretion of urine did not take place. It appeared, however, desirable to repeat the experiment on a larger and less delicate animal, and that, in doing so, it would be right to ascertain whether, under these circumstances, the animal heat was kept up to the natural standard.

Experiment 2.

I repeated the experiment on a middle-sized dog. The temperature of the room was 63° of FAHRENHEIT's thermometer. By having previously secured the carotid and vertebral arteries, I was enabled to remove the head with little or no hæmorrhage. The artificial respirations were made about twenty-four times in a minute. The heart acted with regularity and strength.

At the end of 30 minutes from the time of the spinal marrow being divided, the heart was felt through the ribs contracting 76 times in a minute.

At 35 minutes the pulse had risen to 84 in a minute.

At one hour and 30 minutes the pulse had risen to 88 in a minute.

* Philosophical Transactions for 1809.

At the end of two hours it had fallen to 70, and at the end of two hours and a half to 35 in a minute, and the artificial respiration was no longer continued.

By means of a small thermometer with an exposed bulb, I measured the animal heat at different periods.

At the end of an hour the thermometer in the rectum had fallen from 100° to 94°.

At the end of two hours a small opening being made in the parietes of the thorax, and the ball of the thermometer placed in contact with the heart, the mercury fell to 86°, and half an hour afterwards in the same situation it fell to 78°.

In the beginning of the experiment I made an opening into the abdomen, and having passed a ligature round each artery about two inches below the kidney, brought the edges of the wound in the abdomen together by means of sutures. At the end of the experiment no urine was collected in the ureters above the ligatures.

On examining the blood in the different vessels, it was found of a florid red colour in the arteries, and of a dark colour in the veins, as under ordinary circumstances.

During the first hour and a half of the experiment there were constant and powerful contractions of the muscles of the trunk and extremities, so that the body of the animal was moved in a very remarkable manner, on the table, on which it lay, and twice there was a copious evacuation of faeces.

Experiment 8.

The experiment was repeated on a rabbit. The temperature of the room was 60°. The respirations were made from 30 to 85 in a minute. The actions of the heart at first were

strong and frequent: but at the end of one hour and forty minutes the pulse had fallen to 24 in a minute.

The blood in the arteries was seen of a florid red, and that in the veins of a dark colour.

A small opening was made in the abdominal muscles, through which the thermometer was introduced into the abdomen, and allowed to remain among the viscera.

At the end of an hour the heat in the abdomen had fallen from 100° to 89°. At the end of an hour and forty minutes in the same situation the heat had fallen to 83°, and when the bulb of the thermometer was placed in the thorax in contact with the lungs the mercury fell to 82°.

It has been a very generally received opinion that the heat of warm-blooded animals is dependant on the chemical changes produced on the blood by the air in respiration. In the two last experiments the animals cooled very rapidly, notwithstanding the blood appeared to undergo the usual changes in the lungs, and I was therefore induced to doubt whether the above mentioned opinion respecting the source of animal heat is correct. No positive conclusions however could be deduced from these experiments. If animal heat depends on the changes produced on the blood by the air in respiration, its being kept up to the natural standard, or otherwise, must depend on the quantity of air inspired, and on the quantity of blood passing through the lungs in a given space of time: in other words, it must be in proportion to the fullness and frequency of the pulse, and the fullness and frequency of the inspirations. It therefore became necessary to pay particular attention to these circumstances.

Experiment 4.

The experiment was repeated on a dog of a small size, whose pulse was from 130 to 140 in a minute, and whose respirations, as far as I could judge, were performed from 30 to 35 times in a minute.

The temperature of the room was 65°. The heat in the rectum of the animal at the commencement of the experiment was 99°. The artificial inspirations were made to correspond as nearly as possible to the natural inspirations both in fullness and frequency.

At 20 minutes from the time of the dog being pithed, the heart acted 140 times in a minute with as much strength and regularity as before: the heat in the rectum had fallen to 96½.

At 40 minutes the pulse was still 140 in a minute: the heat in the rectum 92½.

At 55 minutes the pulse was 112, and the heat in the rectum 90°.

At one hour and 10 minutes the pulse beat 90 in a minute, and the heat in the rectum was 88°.

At one hour and 25 minutes the pulse had sunk to 30, and the heat in the rectum was 85°. The bulb of the thermometer being placed in the bag of the pericardium, the mercury stood at 85°, but among the viscera of the abdomen it rose to 87½.

During the experiment there were frequent and violent contractions of the voluntary muscles, and an hour after the experiment was begun, there was an evacuation of fæces.

Experiment 5.

The experiment was repeated on a rabbit, whose respirations, as far as I could judge, were from 30 to 40 in a minute, and whose pulse varied from 130 to 140 in a minute. The

temperature of the room was 57°. The heat in the rectum, at the commencement of the experiment, was 101½. The artificial respirations were made to resemble the natural respirations as much as possible, both in fullness and frequency.

At 15 minutes from the time of the spinal marrow being divided, the heat in the rectum had fallen to 98½°.

At the end of half an hour the heart was felt through the ribs, acting strongly 140 times in a minute.

At 45 minutes the pulse was still 140; the heat in the rectum was 94°.

At the end of an hour the pulse continued 140 in a minute; the heat in the rectum was 92°; among the viscera of the abdomen 94°; in the thorax, between the lungs and pericardium, 92°.

During the experiment, the blood in the femoral artery was seen to be of a bright florid colour, and that in the femoral vein of a dark colour, as usual.

The rabbit voided urine at the commencement of the experiment; at the end of the experiment, no urine was found in the bladder.

Experiment 6.

I procured two rabbits of the same colour, but one of them was about one-fifth smaller than the other. I divided the spinal marrow of the larger rabbit between the occiput and atlas. Having secured the vessels in the neck, and removed the head, I kept up the circulation by means of artificial respiration as in the former experiments. The respirations were made as nearly as possibble similar to natural respirations.

In 23 minutes after the spinal marrow was divided, the pulse was strong, and 130 in a minute: the ball of the thermometer being placed among the viscera of the abdomen, the mercury stood at 96°.

At 34 minutes the pulse was 120 in a minute; the heat in the abdomen was 95°.

At the end of an hour the pulse could not be felt, but on opening the thorax the heart was found acting, but slowly and feebly. The heat in the abdomen was 91°; and between the lobes of the right lung 88°.

During the experiment, the blood in the arteries and veins was seen to have its usual colour.

In this therefore, as in the preceding experiments, the heat of the animal sunk rapidly, notwithstanding the continuance of the respiration. In order to ascertain whether any heat at all was generated by this process, I made the following comparative experiment. The temperature of the room being the same, I killed the smaller rabbit by dividing the spinal marrow between the occiput and atlas. In consequence of the difference of size, *cæteris paribus*, the heat in this rabbit ought to diminish more rapidly than in the other; and I therefore examined its temperature at the end of 52 minutes, considering that this would be at least equivalent to examining that of the larger rabbit at the end of an hour. At 52 minutes from the time of the smaller rabbit being killed, the heat among the viscera of the abdomen was 92°, and between the lobes of the right lung it was 91°. From this experiment, therefore, it appeared not only that no heat was generated in the rabbit, in which the circulation was maintained by artificial respiration, but that it even cooled more rapidly than the dead rabbit.

At the suggestion of Professor Davy, who took an interest in the enquiry, I repeated the foregoing experiment on two animals, taking pains to procure them more nearly of the same size and colour.

Experiment 7.

I procured two large full grown rabbits, of the same colour, and so nearly equal in size, that no difference could be detected by the eye.

The temperature of the room was 57°, and the heat in the rectum of each rabbit previous to the experiment was 100½.

I divided the spinal marrow in one of them, produced artificial respiration, and removed the head after having secured the vessels in the neck. The artificial respirations were made about 35 times in a minute.

During the first hour, the heart contracted 144 times in a minute.

At the end of an hour and a quarter the pulse had fallen to 136 in a minute, and it continued the same at the end of an hour and a half. At the end of an hour and forty minutes the pulse had fallen to 90 in a minute, and the artificial respiration was not continued after this period.

Half an hour after the spinal marrow was divided, the heat in the rectum had fallen to 97°.

At 45 minutes the heat was 95½.

At the end of an hour the heat in the rectum was 94°.

At an hour and a quarter it was 92°.

At an hour and a half it was 91°.

At an hour and forty minutes, the heat in the rectum was 90½, and in the thorax, within the bag of the pericardium, the heat was 87½.

The temperature of the room being the same, the second rabbit was killed by dividing the spinal marrow, and the temperature was examined at corresponding periods.

Half an hour after the rabbit was killed, the heat in the rectum was 99°.

At 45 minutes it had fallen to 98°.

At the end of an hour the heat in the rectum was 96½.

At an hour and a quarter it was 95°.

At an hour and a half it was 94°.

At an hour and forty minutes the heat in the rectum was 93°, and in the bag of the pericardium 90½.

The following table will shew the comparative temperature of the two animals at corresponding periods.

Time.	Rabbit with artificial respiration.		Dead Rabbit.	
	Therm. in the Rectum.	Therm. in the Pericardium.	Thermometer in the Rectum.	Therm. in the Pericardium
Before the Experiment	100½		100½	
30 min.	97		99	
45 —	95½		98	
60 —	94		96½	
75 —	92		95	
90 —	91		94	
100 —	90½	87⅓	93	90½

In this experiment, the thorax, even in the dead animal, cooled more rapidly than the abdomen. This is to be explained by the difference in the bulk of these two parts. The rabbit in which the circulation was maintained by artificial respiration, cooled more rapidly than the dead rabbit, but the difference was more perceptible in the thorax than in the rectum. This is what might be expected, if the production of animal heat does not depend on respiration, since the cold air by which the lungs were inflated, must necessarily have abstracted a certain quantity of heat, particularly as its influence

was communicated to all parts of the body, in consequence of the continuance of respiration.

It was suggested that some animal heat might have been generated, though so small in quantity as not to counterbalance the cooling powers of the air thrown into the lungs. It is difficult or impossible, to ascertain with perfect accuracy, what effect cold air thrown into the lungs would have on the temperature of an animal under the circumstances of the last experiment, independently of any chemical action on the blood: since, if no chemical changes were produced, the circulation could not be maintained, and if the circulation ceased, the cooling properties of the air must be more confined to the thorax, and not communicated in an equal degree to the more distant parts. The following experiment, however, was instituted, as likely to afford a nearer approximation to the truth, than any other that could be devised.

Experiment 8.

I procured two rabbits of the same size and colour: the temperature of the room was 64°. I killed one of them by dividing the spinal marrow, and immediately, having made an opening into the left side of the thorax, I tied a ligature round the base of the heart, so as to stop the circulation. The wound in the skin was closed by a suture. An opening was then made into the trachea, and the apparatus for artificial respiration being fitted into it, the lungs were inflated, and then allowed to collapse as in the former experiment, about 36 times in a minute. This was continued for an hour and a half, and the temperature was examined at different periods. The temperature of the room being the same, I killed the second rabbit in the same manner, and measured the temperature at corresponding periods. The comparative temper-

ature of the two dead animals, under these circumstances, will be seen in the following table.

Time.	Dead Rabbit whose lungs were inflated.		Dead Rabbit whose lungs were not inflated.	
	Therm. in the Rectum.	Therm. in the Thorax.	Therm. in the Rectum.	Therm. in the Thorax
Before the experiment.	100		100	
30 min.	97		98	
45 —	95½		96	
60 —	94		94⅓	
75 —	92½		93	
90 —	91	86	91½	88½

In this last experiment, as may be seen from the above table, the difference in the temperature of the two rabbits, at the end of an hour and a half, in the rectum, was half a degree, and in the thorax two degrees and a half; whereas, in the preceding experiment, at the end of an hour and forty minutes, the difference in the rectum was 2½ degrees, and in the thorax 3 degrees. It appears, therefore, that the rabbit in which the circulation was maintained by artificial respiration, cooled more rapidly on the whole than the rabbit whose lungs were inflated in the same manner after the circulation had ceased. This is what might be expected, if no heat was produced by the chemical action of the air on the blood; since in the last case the cold air was always applied to the same surface, but in the former it was applied always to fresh portions of blood, by which its cooling powers were communicated to the more distant parts of the body.

In the course of the experiments which I have related, I was much indebted to several members of the Society for promoting the Knowledge of Animal Chemistry, for many important suggestions which have assisted me in prosecuting the enquiry.

Mr. HOME, at my request, was present at the seventh experiment. Dr. E. N. BANCROFT was present at, and assisted me in the second experiment; and Mr. WILLIAM BRANDE lent me his assistance in the greater part of those which were made. I have been further assisted in making the experiments by Mr. BROUGHTON, surgeon of the Dorsetshire Regiment of Militia, and Mr. RICHARD RAWLINS, and Mr. ROBERT GATCOMBE, students in Surgery.

I have selected the above from a great number of similar experiments, which it would be needless to detail. It is sufficient to state, that the general results were always the same; and that whether the pulse was frequent or slow, full or small, or whether the respirations were frequent or otherwise, there was no perceptible difference in the cooling of the animal.

From the whole we may deduce the following conclusions:

1. The influence of the brain is not directly necessary to the action of the heart.

2. When the brain is injured or removed, the action of the heart ceases, only because respiration is under its influence, and if under these circumstances respiration is artificially produced, the circulation will still continue.

3. When the influence of the brain is cut off, the secretion of urine appears to cease, and no heat is generated; notwithstanding the functions of respiration, and the circulation of the blood continue to be performed, and the usual changes in the appearance of the blood are produced in the lungs.

4. When the air respired is colder than the natural temperature of the animal, the effect of respiration is not to generate, but to diminish animal heat

On the Expansion of any Functions of Multinomials. By Thomas Knight, *Esq.* *Communicated by* Humphry Davy, *Esq. LL.D. Sec. R. S.*

Read June 7th, 1810.

1. The expansion of multinomial functions has, of late, been so ably and fully treated by M. Arbogast, in his learned work '*Du Calcul des Dérivations*,' that it may appear, perhaps, scarcely necessary to add any thing to what has been written, on the subject, by that excellent geometer.

Nevertheless, as he is the only one that has hitherto cultivated this part of analysis with any *great* success; and as it is agreeable, I believe, to most persons, to be presented with various solutions to mathematical problems, I hope it will not be thought superfluous if I show how the same things may be accomplished in a very different manner.

By the procedure here made use of, we shall also be enabled to arrive at many new and remarkable theorems (both for *direct* and *inverse derivation*), which could not, I imagine, be very easily found by M. Arbogast's methods.

For a function of *one* simple multinomial, I give (amongst others) the same rules of *direct derivation*, as that author; but when there are *many*, and in the more difficult cases of double and triple multinomials, &c. or functions of any number of these, I offer new and expeditious methods; which are demonstrated with the less trouble, from the analogy which

reigns throughout, in this manner of treating the subject; and the regularity with which we proceed from the easy to the more complex cases. By means of this analogy also, the reader may without difficulty keep all the rules in his memory.

2. I shall begin with *the expansion of any function of a simple multinomial.*

*First method.** If $f(c+z)$ represent any function of $c+z$, and the fluxions be taken, separately, with respect to c and z, the fluxional coefficient is the same in both cases: or $\left(\frac{\dot{f}(c+z)}{\dot{c}}\right) = \left(\frac{\dot{f}(c+z)}{\dot{z}}\right)$; whence it follows, that $\int\left(\frac{\dot{f}(c+z)}{\dot{c}}\right)\dot{z} = \int\left(\frac{\dot{f}(c+z)}{\dot{z}}\right)\dot{z} = f(c+z)$. This being premised, let

$$f(c+\dot{c}x+\ddot{c}x^2+\dddot{c}x^3+) = B+\dot{B}x+\ddot{B}x^2+\dddot{B}x^3+\ldots\ldots$$
$$+B^{n}x^{n}+, \text{ \&c. } \ldots\ldots (1);$$

let $\dot{B}, \ddot{B}, \dddot{B}$, &c. represent the fluxional coefficients of B, B, B, &c. with respect to c, and we shall have

$$\left(\frac{\dot{f}(c+\dot{c}x+\ddot{c}x^2+\dddot{c}x^3+)}{\dot{c}}\right) = \dot{B}+\ddot{B}x+\dddot{B}x^2+\dot{B}x^3+\ldots\ldots+\dot{B}^{n}x^{n}+.$$

If we multiply this by $\dot{c}\dot{x}+2\ddot{c}x\dot{x}+3\dddot{c}x^2\dot{x}+\ldots\ldots+nc\,x^{n-1}\dot{x}+$ and take the fluent, we shall get, by what was just now shewn, another expansion of $f(c+\dot{c}x+\ddot{c}x^2+\dddot{c}x^3+)$;

* See LA CROIX, '*Traité élément. de Calc. Différent.*' p. 25, note; where a similar proceeding is used for binomial functions.

and by comparing the coefficients, of the different powers of x, with those in equation (1), there will be found,

$$B = \overset{'}{c}\,\overset{'}{B}$$

$$B = \frac{2\,\overset{''}{c}\,\overset{'}{B} + \overset{'}{c}\,\overset{1}{B}}{2}$$

$$B = \frac{3\,\overset{''}{c}\,\overset{1}{B} + 3\,\overset{''}{c}\,\overset{2}{B} + \overset{'}{c}\,\overset{3}{B}}{3}$$

$$B = \frac{n\,\overset{''\ldots n}{c}\,\overset{'}{B} + (n-1)\,\overset{''\ldots(n-1)}{c}\,\overset{1}{B} + (n-2)\,\overset{''\ldots(n-2)}{c}\,\overset{2}{B} + \ldots + 2\,\overset{''}{c}\,\overset{n-2}{B} + \overset{'}{c}\,\overset{n-1}{B}}{n}$$

..... (2).

But $B = f(c)$, $\overset{'}{B} = \overset{'}{f}(c)$, whence all the rest are known. I represent by strokes over the f the fluxional coefficients of $f(c)$; the number of strokes marking the order,

Though this is a complete solution of the problem, it affords by no means an easy way of calculating the coefficients; on which account I shall not trouble the reader with examples. It will be shewn presently, that the method of derivation in M. Arbogast's first section is easily obtained from this.

3. *Second Method.* I here, as in the former case, consider the quantity $c + \overset{'}{c}\,x + \overset{''}{c}\,x^2 + \overset{'''}{c}\,x^3 +$ to be a binomial, and take the fluxional coefficient of the function with respect to c; but multiply by the partial fluxion $\dot{c}\,x$, instead of $\dot{c}\,\dot{x} + 2\,\dot{c}\,x\,\dot{x}$ +, &c.; we find, by this way of proceeding, $\int \overset{B}{n-1}\,\overset{'}{c}$ for the sum of all those terms in $\overset{B}{n}$ that are multiplied by the powers

of c. In like manner, $\int \overset{B}{}_{n-2}\, \overset{''}{c}$ will be the sum of all those terms, in the same coefficient, that contain $\overset{''}{c}$ and its powers; and, in general, $\int \overset{B}{}_{n-m}\, \overset{''\ldots m}{c}$ the sum of those that have for factors the powers of $\overset{''\ldots m}{c}$.

Hence is derived an easy method of finding any coefficient, when we know those that precede it: for if these partial values be united, there arises

$$\overset{B}{}_{n} = \int \overset{B}{}_{n-1}\, \overset{'}{c} + \int \overset{B}{}_{n-2}\, \overset{''}{c} + \int \overset{B}{}_{n-3}\, \overset{'''}{c} +, \text{ \&c. ..... } (9),$$

provided that we neglect

in $\overset{B}{}_{n-2}$ all those terms which contain $\overset{'}{c}$,

in $\overset{B}{}_{n-3}$ all those which contain $\overset{'}{c}$ or $\overset{''}{c}$,

in $\overset{B}{}_{n-4}$ all those which have $\overset{'}{c}$ or $\overset{''}{c}$ or $\overset{'''}{c}$,

and so on; whence it happens, that many of the B's will be neglected entirely, and the chief part of the operation will always be in the first term $\int \overset{B}{}_{n-1}\, \overset{'}{c}$. From equation (9), we find the first part of the expansion of $f(c + \overset{'}{c}x + \overset{''}{c}x^2 +)$ to be

$$\begin{array}{l|l|l|l|l}
f(c) + f'(c)\overset{\cdot}{c} & + f'(c)\overset{'}{c}x + f'(c)\overset{''}{c} & + f'(c)\overset{'''}{c} & + f'(c)\overset{''''}{c} & \\
\quad + f''(c)\frac{\overset{\cdot}{c}^2}{2} & \quad + f''(c)\overset{'}{c}\overset{''}{c} & \quad + f''(c)\left\{\overset{'}{c}\overset{'''}{c} + \frac{\overset{''}{c}^2}{2}\right\} & \quad + f''(c)\left\{\overset{'}{c}\overset{''''}{c} + \overset{'}{c}\frac{\overset{'''}{c}}{2}\right\} & \\
 & \quad + f'''(c)\frac{\overset{'}{c}^2}{2\cdot 3} & \quad + f'''(c)\frac{\overset{\cdot}{c}^2}{2}\overset{''}{c} & \quad + f'''(c)\frac{\overset{\cdot}{c}^2}{2}\overset{''}{c} & \\
 & & \quad + f''''(c)\frac{\overset{'}{c}^3}{2\cdot 3\cdot 4} & \quad + f''''(c)\frac{\overset{'}{c}^2}{2}\overset{''}{c} & \\
 & & & \quad + f'''''(c)\frac{\overset{'}{c}^3}{2\cdot 3\cdot 4\cdot 5} &
\end{array} x^5 +$$

But, that we may enter rather more into particulars, let it be required from the terms already given, to find $\overset{B}{6}$ the coefficient of x^6.

To make the operation plain, I have put a star over every term we are to use, excepting the coefficient of x^3, which is wholly employed.

$$\int \overset{\dot{B}}{6-1}\dot{c} = \int \overset{\dot{B}}{5}\dot{c} = \dot{f}(c)\dot{c}\overset{\dddot{*}}{c} + \overset{\dddot{*}}{f}(c)\left|\begin{matrix}\frac{\dot{c}^2}{2}\overset{\dddot{*}}{c} + \overset{\dddot{*}}{f}(c)\\ +\dot{c}\overset{\ddot{*}}{c}\overset{\dddot{*}}{c}\end{matrix}\right|\left|\begin{matrix}\frac{\dot{c}^3}{2.3}\overset{\dddot{*}}{c} + \overset{\dddot{*}}{f}(c)\frac{\dot{c}^4}{2.3.4}\overset{\ddot{*}}{c} + \overset{\dddot{*}}{f}(c)\frac{\dot{c}^5}{2.3.4.5}\\ +\frac{\dot{c}^2\ddot{c}^2}{2.2}\end{matrix}\right|;$$

$$\int \overset{\dot{B}}{4}\ddot{c} = \ddot{f}(c)\ddot{c}\overset{\dddot{*}}{c} + \overset{\dddot{*}}{f}(c)\frac{\ddot{c}^3}{2.3}; \int \overset{\dot{B}}{3}\overset{\dddot{*}}{c} = \overset{\ddot{*}}{f}(c)\frac{\overset{\dddot{*}}{c^3}}{2}; \int \overset{\dot{B}}{B}\overset{\dddot{*}}{c}\overset{6}{} = \dot{f}(c)\overset{\dddot{*}}{c}\overset{6}{};$$

and by adding these together, we get

$$\overset{B}{6} = \dot{f}(c)\overset{\dddot{*}}{c}\overset{6}{} + \ddot{f}(c)\left|\begin{matrix}\dot{c}\overset{\dddot{*}}{c}\overset{\dddot{*}}{c} + \overset{\dddot{*}}{f}(c)\\ +\dot{c}\overset{\dddot{*}}{c}\overset{\dddot{*}}{c}\\ +\frac{\overset{\dddot{*}}{c^3}}{2}\end{matrix}\right|\left|\begin{matrix}\frac{\dot{c}^2}{2}\overset{\dddot{*}}{c} + \overset{\dddot{*}}{f}(c)\\ +\dot{c}\ddot{c}\overset{\dddot{*}}{c}\\ +\frac{\ddot{c}^3}{2.3}\end{matrix}\right|\left|\begin{matrix}\frac{\dot{c}^3}{2.3}\overset{\dddot{*}}{c} + \overset{\dddot{*}}{f}(c)\frac{\dot{c}^4}{2.3.4}\ddot{c} + \overset{\dddot{*}}{f}(c)\frac{\dot{c}^5}{2.3.4.5}\\ +\frac{\dot{c}^2\ddot{c}^2}{2.2}\end{matrix}\right|.$$

This process is sufficiently easy; but, in order to find any coefficient as $\overset{B}{n}$, it is by no means necessary for us to know all those that precede it; it may be immediately obtained from $\overset{B}{n-1}$ by a variety of ways: but we must first learn how to express $\overset{\dot{B}}{n-m}$ by the fluxional coefficients of $\overset{B}{n-1}$; after which we shall only have to substitute in equations (2) and (3).

Now it appears, from what has been shewn in this article, that

$$\overset{\dot{B}}{n-1} = \left(\frac{\overset{\dot{B}}{n}}{\dot{c}}\right); \overset{\dot{B}}{n-2} = \left(\frac{\overset{\dot{B}}{n}}{\ddot{c}}\right) = \left(\frac{\overset{\dot{B}}{n-1}}{\dot{c}}\right); \overset{\dot{B}}{n-3} = \left(\frac{\overset{\dot{B}}{n}}{\overset{\dddot{*}}{c}}\right) = \left(\frac{\overset{\dot{B}}{n-1}}{\ddot{c}}\right)$$

$$= \left(\frac{\overset{\dot{B}}{n-2}}{\dot{c}}\right) = \left(\frac{\overset{\ddot{B}}{n-1}}{\dot{c}^2}\right);$$ (where by strokes put *under* a quan-

tity, I represent the reverse of the operations denoted by the strokes *over* it)* and, in general,

$$\overset{\acute{B}}{n-m} = \left(\frac{\overset{\acute{B}}{n}}{\overset{\prime\ldots m}{c}}\right) = \left(\frac{\overset{\acute{B}}{n-1}}{\overset{\prime\ldots(m-1)}{c}}\right) =, \text{ \&c. } \ldots(4); \quad \overset{\acute{B}}{n-m} = \left(\frac{\overset{m-1}{\acute{B}}}{\overset{\prime\ldots m-1}{c}}\right)_{n\ldots\ldots(m-1)} \quad \ldots\ldots(5).$$

It is evident that we might find many other relations between the B's and their fluxional coefficients; but those I have given seem the most useful.

4. By means of these equations, we may find $\overset{B}{n}$ from $\overset{B}{n-1}$ in several ways.

First Method. If we substitute, in equation (2), for $\overset{\acute{B}}{n-2}$, $\left(\frac{\overset{\acute{B}}{n-1}}{\overset{\prime}{c}}\right)$; for $\overset{\acute{B}}{n-3}$, $\left(\frac{\overset{\acute{B}}{n-1}}{\overset{\prime\prime}{c}}\right)$; for $\overset{\acute{B}}{n-4}$, $\left(\frac{\overset{\acute{B}}{n-1}}{\overset{\prime\prime\prime}{c}}\right)$; ..... for $\overset{\acute{B}}{n-n}$, $\left(\frac{\overset{\acute{B}}{n-1}}{\overset{\prime\ldots n(-1)}{c}}\right)$; which values are got from equation (4), we find

$$n\overset{B}{n} = n\overset{n\ldots n}{c}\left(\frac{\overset{\acute{B}}{n-1}}{\overset{\prime\ldots(n-1)}{c}}\right) + (n-1)\overset{\prime\ldots(n-1)}{c}\left(\frac{\overset{\acute{B}}{n-1}}{\overset{\prime\ldots(n-2)}{c}}\right) + \ldots\ldots\ldots$$

$$+ 2\overset{n}{c}\left(\frac{\overset{\acute{B}}{n-1}}{\overset{\prime}{c}}\right) + \overset{\prime}{c}\left(\frac{\overset{\acute{B}}{n-1}}{\overset{\prime}{c}}\right) \ldots\ldots (6).$$

This expression agrees with M. ARBOGAST's first method, and affords the following rule.

To find $\overset{B}{n}$ *from* $\overset{B}{n-1}$, *take the fluxion of the latter, with respect to* $\dot{c}, \overset{\prime}{c}, \overset{\prime\prime}{c}$, *&c. and change generally* $\overset{\prime\ldots m}{c}$ *into* $\overset{\prime\ldots(m+1)}{c} \times \frac{m+1}{n}$.

* Any number of strokes under a quantity will represent the depression of the fluxional coefficients of + (2) therein contained so many orders.

This rule is, however, more simple in the enunciation than the practice; on which account I proceed to a

5. *Second Method.* We might obtain one from equations (2) and (3), but, as it would be somewhat worse than the last, I omit it; and substitute in equation (3), the values of $\overset{\text{B}}{}_{n-2}$, $\overset{\text{B}}{}_{n-3}$, &c. given by (5). We find thus

$$\overset{\text{B}}{}_{n} = \int \left(\frac{\overset{\text{B}}{}_{n-1}}{\dot{c}}\right)\dot{c} + \int \left(\frac{\overset{\text{B}}{}_{n-1}}{\dot{c}}\right)''c + \int \left(\frac{\overset{\text{B}}{}_{n-1}}{\dot{c}}\right)'''c + \int \left(\frac{\overset{\text{B}}{}_{n-1}}{\dot{c}}\right)''''c + \text{, \&c. .... (7),}$$

or, if we consider $\overset{\text{B}}{}_{n-1}$ under the form $\overset{\text{B}}{}_{n-1} = \beta + \overset{(1)}{\beta}\dot{c} + \overset{(2)}{\beta}\ddot{c} + \overset{(3)}{\beta}\dddot{c} +$,

$$\overset{\text{B}}{}_{n} = \int \overset{\text{B}}{}_{n-1}\dot{c} + 1\int \overset{(1)}{\beta}\ddot{c} + 1.2\int \overset{(2)}{\beta}\dddot{c} + 1.2.3\int \overset{(3)}{\beta}\ddot{c} + \text{, \&c. .....(8)}^{*}$$

Where any number of strokes under the β's denotes that the fluxional coefficients of $f(c)$ therein contained, must be depressed so many orders.

In this expression, we must neglect in $\overset{(5)}{\beta}$ all terms containing $\ddot{c}$, in $\overset{(3)}{\beta}$ all those containing $\dddot{c}$ or $\ddot{c}$, and so on. Let it be required, for an example, to find the coefficient of x^{7}, from that of x^{6} given in article 3. We shall have, after neglecting such terms as are above specified,

$$\overset{(1)}{\beta} = f'(c)\dddot{c} + f''(c)\dot{c}\ddot{c}; \quad \overset{(2)}{\beta} = f''(c)\frac{\dddot{c}}{2}; \quad \overset{(6)}{\beta} = f^{\text{IV}}(c)\frac{1}{2.3.4.5.6},$$

and by performing the operations indicated by equation (8), we find

$$\int \overset{\text{B}}{}_{6}\dot{c} = f'(c)\dot{c}\ddot{c}\dddot{c} + f''(c)\left|\begin{matrix}\frac{\dot{c}^{3}}{2}\dddot{c} \\ +\dot{c}\ddot{c}\dddot{c} \\ +\dot{c}\frac{\ddot{c}^{3}}{2}\end{matrix}\right| + f'''(c)\left|\begin{matrix}\frac{\dot{c}^{3}}{2.3}\dddot{c} \\ +\frac{\dot{c}^{2}}{2}\ddot{c}\dddot{c} \\ +\dot{c}\frac{\ddot{c}^{3}}{2.3}\end{matrix}\right| + f^{\text{IV}}(c)\left|\begin{matrix}\frac{\dot{c}^{4}}{2.3.4}\dddot{c} \\ +\frac{\dot{c}^{3}}{2.3}\frac{\ddot{c}^{2}}{2}\end{matrix}\right| + f^{\text{V}}(c)\frac{\dot{c}^{5}}{2.3.4.5}\ddot{c} + f^{\text{VI}}(c)\frac{\dot{c}^{6}}{2.3.4.5.6} + f^{\text{VII}}(c)\frac{\dot{c}^{7}}{2.3.4.\ldots.7}$$

* See Note III. at the end.

$\int \overset{(1)}{\beta}\, \overset{''}{c} = \dot{f}(c)\, \overset{''}{c}\, \overset{'''}{c} + \overset{''}{f}(c)\, \frac{\overset{''}{c^2}}{2}\, \overset{'''}{c}$; $2\int \overset{(2)}{\beta}\, \overset{'''}{c} = \dot{f}(c)\, \overset{''}{c}\, \overset{'''}{c}$;

$2.3.4.5.6 \int \overset{(6)}{\beta}\, \overset{''\ldots 7}{c} = \dot{f}(c)\, \overset{''\ldots 7}{c}$; these being added give $\overset{B}{7}$ the coefficient of x', which was required.

To find $\overset{B}{n}$ from equation (7) requires the use of both fluxions and fluents; in (8) we are without the fluxional process; but, in its place, have the trouble of observing the numeral coefficients of each term: there is, however, a way of avoiding the mention of fluents, and the necessity of paying attention to these coefficients. If we consider equation (8) and the mode of expansion derived from it, it will be evident, that whenever we have any power (as the *m*th) of $\dot{c}$ or $\overset{''}{c}$ or $\overset{'''}{c}$, or &c. it must be divided by the product 2.3.4....*m*. This follows from the manner of finding the fluent of such a quantity as $\overset{r}{c}\, \dot{c}$, and the consideration that we cannot arrive at $\overset{m}{c}$ without having passed through all the lower powers, and repeated the fluential process at each. Hence results the following rule; (where by $\overset{(2)}{\beta}$, $\overset{(3)}{\beta}$, &c. I mean these quantities after we have neglected in each of them the terms that have been already specified).

Omitting all the denominators, multiply $\overset{B}{n-1}$ *by* $\dot{c}$; $\overset{(1)}{\beta}$ *by* $\overset{''}{c}$; $\overset{(2)}{\beta}$ *by* $\overset{'''}{c}$; $\overset{(3)}{\beta}$ *by* $\overset{''''}{c}$; *and so on: add these products together, and wherever there is any power of* $\dot{c}$, $\overset{''}{c}$, *&c. as the* m*th, put the product* 2.3.4....*m for a denominator.*

6. *Third Method.* If we substitute in equation (3) the

values of $\overset{\text{B}}{\underset{n-2}{}}$, $\overset{\text{B}}{\underset{n-3}{}}$, &c. given by (4), and it will become

$$\text{B}_n = \int\left(\frac{\overset{\text{B}}{\underset{n-1}{}}}{\dot{c}}\right)\dot{c}\cdot + \int\left(\frac{\overset{\text{B}}{\underset{n-1}{}}}{\dot{c}}\right)''c\cdot + \int\left(\frac{\overset{\text{B}}{\underset{n-1}{}}}{\dot{c}}\right)'''c\cdot + \int\left(\frac{\overset{\text{B}}{\underset{n-1}{}}}{\dot{c}}\right)''''c\cdot +, \text{ \&c. ..... (9)},$$

where we must neglect, in the second term, every thing that contains $\dot{c}$; in the third term every thing that contains $\dot{c}$ or $''c$; and so on.

If we did not do this, we should have the same combinations of letters, frequently, more than once. We may, however, instead of proceeding according to the above given direction, omit the superfluous terms *at last*; and then the rule will be as follows:

To find $\overset{\text{B}}{\underset{n}{}}$, *take the fluxion of* $\overset{\text{B}}{\underset{n-1}{}}$ *with respect to* $c, \dot{c}, ''c$, *&c. and after changing, every where,* $c^{''\ldots m}\cdot$ *into* $c^{''\ldots(m+1)}$, *take the fluent with respect to this last; observing to keep only once the same combination of letters.*

But now let us consider, whether we cannot, by omitting to make certain of the letters vary, prevent the same combinations from being repeated.

First, if, in the term $\text{P}\times\left(c^{''\ldots(r-m)}\right)^n\times\left(c^{''\ldots r}\right)^q$, we make $c^{''\ldots(r-m)}$ vary, according to the rule just now given, there results the combination $\text{P}\times\left(c^{''\ldots(r-m)}\right)^{n-1}\times c^{''\ldots(r-m+1)}\times\left(c^{''\ldots r}\right)^q$ .... (a); but the same fluxional coefficient of $f(c)$ that is multiplied by $\text{P}\times\left(c^{''\ldots(r-m)}\right)^n\times\left(c^{''\ldots r}\right)^q$ will be also multiplied by $\text{P}\times\left(c^{''\ldots(r-m)}\right)^{n-1}$

$\times c^{\prime\prime\ldots(r-m+1)} \times c^{\prime\prime\ldots(r-1)} \times \left(c^{\prime\prime\ldots r}\right)^{q-1}$, (where $c^{\prime\prime\ldots(r-1)}$ is either the last factor, with respect to the number of strokes, or the last but one, accordingly as q is equal to or greater than one); if we make this term vary, with respect to $c^{\prime\prime\ldots(r-1)}$, we shall have the combination marked (α) over again.

Let us next consider when it will be necessary to make c vary. In the term $f^{\prime\prime\ldots n}(c) \times P \times \left(c^{\prime\prime\ldots m}\right)^p$ ..... (β), if we make c vary, there arises the combination $f^{\prime\prime\ldots(n+1)}(c) \times c' \times P \times \left(c^{\prime\prime\ldots m}\right)^p$; but the same coefficient (as B) that contains (β), will also contain $f^{\prime\prime\ldots(n+1)}(c) \times c' \times P \times c^{\prime\prime\ldots(m-1)} \times \left(c^{\prime\prime\ldots m}\right)^{p-1}$ which when we make it vary with respect to $c^{\prime\prime\ldots(m-1)}$ gives also $f^{\prime\prime\ldots(n+1)}(c) \times c' \times P \times \left(c^{\prime\prime\ldots m}\right)^p$. Now $c^{\prime\prime\ldots(m-1)}$ was here the last quantity or the last but one. We may then affirm, in general, that, if we make every term in B^{n-1} vary with respect to the last quantity, and the last but one also, when this immediately precedes the last, not in place only, but in the number of its strokes, we shall get all the terms we ought to have, any further variation only giving the same over again. From equation (9) we have then the following

Rule.

To find B^n *take the fluxion of* B^{n-1} *with respect to the last of the quantities* $c, c', c'', \&c.$ *in each term, and the last but one also, if it immediately precede the last in the number of strokes.*

Change every where $c^{\prime\prime\ldots m}$ *into* $c^{\prime\prime\ldots(m+1)}$ *and take the fluent with respect to this last.*

This is exactly the same rule as that given by M. ARBOGAST in p. 25 of his work.

7. The method pursued in this paper, has a remarkable advantage over M. ARBOGAST's in what he calls *inverse derivation*;* which I shall shew hereafter to be extremely useful in the expansion of double and triple, &c. multinomials. In the present case, of a simple one, we have, as was shewn at the end of Art. 3,

$$\mathrm{B}_{n-1} = \left(\frac{\dot{\mathrm{B}}_{n}}{c'}\right)_{1}; \quad \mathrm{B}_{n-2} = \left(\frac{\dot{\mathrm{B}}_{n}}{c''}\right)_{1}; \quad \mathrm{B}_{n-3} = \left(\frac{\dot{\mathrm{B}}_{n}}{c'''}\right)_{1}; \quad \ldots\ldots \mathrm{B}_{n-m} = \left(\frac{\dot{\mathrm{B}}_{n}}{c^{\prime\prime\ldots m}}\right)_{1};$$

whence this

Rule.

To find B_{n-m}, *the coefficient of* x^{n-m}, *from* B_{n} *that of* x^{n}, *take the fluxional coefficient of the latter, with respect to* $c^{\prime\prime\ldots m}$, *and at the same time depress, to the next lower order, all the fluxional coefficients of* $f(c)$ *that are in* $\left(\frac{\dot{\mathrm{B}}_{n}}{c^{\prime\prime\ldots m}}\right)$.

Thus from the coefficient of x^{5}, which was found in Art. 3, we get

$$\mathrm{B}_{5} = \left(\frac{\dot{\mathrm{B}}_{6}}{c'}\right)_{1} = \dot{f}(c)\,c''' + f''(c)\left|\begin{matrix} c'\,c''' \\ + c''\,c''' \end{matrix}\right. + f'''(c)\left|\begin{matrix} \frac{c'^{2}}{2}\,c''' \\ + c'\frac{c''^{2}}{2} \end{matrix}\right. + f^{\mathrm{IV}}(c)\,\frac{c'^{2}}{2.3}\,c'' + f^{\mathrm{V}}(c)\,\frac{c'^{5}}{2.3.4.5}$$

* See Note III. at the end.

$$\mathrm{B}_4 = \left(\begin{matrix} \frac{b}{6} \\ \frac{n}{c^n} \end{matrix}\right)_1 = f(c)\,\overset{m}{c} + f'(c)\left|\begin{matrix} \overset{'}{c}\,\overset{m}{c} + \overset{m}{f}(c)\,\frac{\overset{'}{c}^2}{2}\,\overset{n}{c} + \overset{m}{f}(c)\,\frac{\overset{'}{c}^3}{2.3.4} \\ + \frac{\overset{n}{c}^2}{2} \end{matrix}\right.$$

&c. &c.

8. To complete the theory of the expansion of any function of a simple multinomial, there remains, for us to solve, the following

Problem.

It is required to find B_n *without knowing any of the coefficients that precede or follow it.*

It is, in the first place, evident enough, from what has been done, that

$$\mathrm{B}_n = f(c)\,\overset{n}{c} + f'(c)\,\overset{n}{\Psi} + \overset{m}{f}(c)\,\overset{m}{\Psi} + \ldots\ldots\ldots + \overset{n\ldots(m-1)}{f}(c)\,\overset{n\ldots(m-1)}{\Psi}$$

$$+ \overset{n\ldots m}{f}(c)\,\overset{n\ldots m}{\Psi} + \ldots\ldots\ldots + \overset{n\ldots n}{f}(c)\,\frac{\overset{'n}{c}}{2.3.4\ldots n},$$

where $\overset{n\ldots m}{\Psi}$ consists (without considering the denominators) of all the combinations that can be formed of $\overset{'}{c}, \overset{''}{c}, \overset{m}{c}$, &c. in which the sum of the strokes shall be n,* and the sum of the exponents m. But to form these combinations, for the higher powers, would not be very easy. It may not be amiss to inquire, therefore, for some regular method of immediately deriving $\overset{n\ldots(m-1)}{\Psi}$ from $\overset{n\ldots m}{\Psi}$; so that we may get all the Ψ's successively, beginning with $\frac{\overset{'n}{c}}{2.3\ldots n}$ which multiplies $f(c)$.

* I mean when the powers are expanded, as when c^3 is written $\overset{'}{c}\,\overset{'}{c}\,\overset{'}{c}$.

I shall take no notice of any numbers, which divide the different terms, till the end of the operation; having shewn, in Article 5, that it will be sufficient then to place the product $2.3.4....\mu$ under every μth power.

There can be no difficulty in perceiving, that all the combinations in $\psi^{\prime\prime\ldots(m-1)}$ may be derived from those terms in $\psi^{\prime\prime\ldots m}$, that are multiplied by the powers of c', in the following manner. First, diminish the exponent of c' by one. Then, diminish the exponent of one of the other quantities by one, and multiply by the quantity that has the next greater number of strokes. For, if $\left(c^{\prime\prime\ldots r}\right)^{a} \times P \times \left(c^{\prime\prime\ldots s}\right)^{b}$ be one of the combinations in $\psi^{\prime\prime\ldots(m-1)}$, there must necessarily be in $\psi^{\prime\prime\ldots m}$ the combination $c' \times \left(c^{\prime\prime\ldots r}\right)^{a} \times P \times c^{\prime\prime\ldots(s-1)} \times \left(c^{\prime\prime\ldots s}\right)^{b-1}$; and from this the former one is derived, in the manner above-mentioned, by taking away the c' and changing $c^{\prime\prime\ldots(s-1)}$ into $c^{\prime\prime\ldots s}$.

We will next see if there be any quantities that it would be superfluous to make vary. Let $c^{b} \times P \times \left(c^{\prime\prime\ldots(p-q)}\right)^{r} \times \left(c^{\prime\prime\ldots p}\right)^{s}$ be a term of $\psi^{\prime\prime\ldots m}$, we find from it, by the prescribed operation, the two terms of $\psi^{\prime\prime\ldots(m-1)}$

$c^{b-1} \times P \times \left(c^{\prime\prime\ldots(p-q)}\right)^{r} \times \left(c^{\prime\prime\ldots p}\right)^{s-1} \times c^{\prime\prime\ldots(p+1)}$ and $c^{b-1} \times P \times \left(c^{\prime\prime\ldots(p-q)}\right)^{r-1} \times c^{\prime\prime\ldots(p-q+1)} \times \left(c^{\prime\prime\ldots p}\right)^{s}$.... ($\alpha$); but $\psi^{\prime\prime\ldots m}$ will also have the combination $c^{b} \times P \times \left(c^{\prime\prime\ldots(p-q)}\right)^{r-1} \times c^{\prime\prime\ldots(p-q+1)} \times c^{\prime\prime\ldots(p-1)} \times \left(c^{\prime\prime\ldots p}\right)^{s-1}$, from which the combination (α) may be got by diminishing by

one the exponent of c and changing $c^{\prime\ldots(p-1)}$ into $c^{\prime\ldots p}$. Now $c^{\prime\ldots(p-1)}$ *is either the last quantity or the last but one in the order of the strokes.*

We can then have no difficulty in perceiving the truth of the following

*Rule.**

1st. To find $\downarrow^{\prime\ldots(m-1)}$ *take no notice of any terms in* $\downarrow^{\prime\ldots m}$ *but those that are multiplied by the powers of* c, *in all these diminish the exponent of* c *by one; and omit the denominators.*

2dly. Diminish the exponent of the last quantity, in these terms, by one; and multiply by the quantity that has the next greater number of strokes.

3dly. If the last quantity but one be that which immediately precedes the last in the number of strokes, make it vary in the same manner as was directed for the last.

4thly. All the combinations being thus formed, put the product $2.3.4....\mu$ *under every* μ*th power.*

The reader may compare this rule with that given by M. ARBOGAST, p. 36.

Suppose it were required to find B; we must begin with

$\downarrow^{\prime\ldots 10}_{10} = \frac{c^{10}}{2.3.4...10}$; from which is derived by the rule, $\downarrow^{\prime\ldots 9} = \frac{c^{8}}{2.3...8} \times c^{\prime}$,

$\downarrow^{\prime\ldots 8} = \frac{c^{7}}{2.3...7} \times c^{\prime\prime} + \frac{c^{6}}{2.3...6} \times \frac{c^{2}}{2}$, and so on, whence

$$B = f(c)\frac{c^{10}}{2.3.4...10} + f(c)\frac{c^{8}}{2.3.4...8}c^{\prime\prime} + f(c)\left\{\frac{c^{7}}{2.3.4...7}c^{\prime\prime\prime} + \frac{c^{6}}{2.3.4...6}\frac{c^{2}}{2}\right\}$$

$+$, &c.

* See Note I. at the end.

The succeeding terms are found with equal ease, I omit to find them only on account of the length of the calculation.

9. I shall now show that the same method may be successfully employed in more complicated cases; and, instead of dwelling on particular problems, shall proceed at once to *the expansion of any function of any functions of simple multinomials*,

$$\varphi\left\{F(c+\overset{'}{c}x+\overset{''}{c}x^2+),\ f(e+\overset{'}{e}x+\overset{''}{e}x^2+),\ \&c.\right\}\ldots\ldots(\alpha).$$

If we consider $c+\overset{'}{c}x+$, $e+\overset{'}{e}x+$, &c. as binomials $c+y$, $e+z$, &c. the function (α), which may be, for the moment, represented by ϕ, will have for its fluxion, (y, z, &c. being made to vary)

$$\left(\frac{\phi}{y}\right)\dot{y}+\left(\frac{\phi}{z}\right)\dot{z}+\&c.=\left(\frac{\phi}{c}\right)\dot{y}+\left(\frac{\phi}{e}\right)\dot{z}+\&c.;$$ and consequently

$$\int\left\{\left(\frac{\phi}{c}\right)\dot{y}+\left(\frac{\phi}{e}\right)\dot{z}+\&c.\right\}=\phi=\varphi\left\{F(c+\overset{'}{c}x+\overset{''}{c}x^2+),\ f(e+\overset{'}{e}x+\overset{''}{e}x^2+),\ \&c.\right\}\ldots(\beta).$$

If then we represent the expansion of the function (α) by the series

$$B+\underset{1}{B}x+\underset{2}{B}x^2+\underset{3}{B}x^3+\ldots\ldots+\underset{n}{B}x^n+\ldots\ldots(\gamma)$$

and denote the fluxional coefficients, of the first order of $\underset{1}{B}$, $\underset{2}{B}$, $\underset{3}{B}$, &c. with respect to c thus $\overset{c}{\underset{1}{B}}$, $\overset{c}{\underset{2}{B}}$, $\overset{c}{\underset{3}{B}}$, &c.;

with respect to e thus $\overset{e}{\underset{1}{B}}$, $\overset{e}{\underset{2}{B}}$, $\overset{e}{\underset{3}{B}}$, &c.;

&c. &c.

the equation marked (β) will become

$$\varphi\left\{F(c+\overset{'}{c}x+),\ f(e+\overset{'}{e}x+),\ \&c.\right\}=\int\left\{\overset{c}{B}+\overset{c}{\underset{1}{B}}x+\overset{c}{\underset{2}{B}}x^2+\right\}$$

$$(\dot{c}\dot{x}+2\ddot{c}x\dot{x}+)+\int\left\{\overset{e}{\mathrm{B}}+\overset{e}{\underset{1}{\mathrm{B}}}x+\overset{e}{\underset{2}{\mathrm{B}}}x^2+\right\}(\dot{e}\dot{x}+2\ddot{e}x\dot{x}+)+\&c.$$

whence, after taking the fluents, and comparing the coefficients of the powers of x, with those of the same powers in (γ), we find

$$\mathrm{B}=\dot{c}\overset{e}{\mathrm{B}}+\dot{e}\overset{e}{\mathrm{B}}+\&c.$$

$$\mathrm{B}=\frac{2\ddot{c}\overset{c}{\mathrm{B}}+\dot{c}\overset{c}{\underset{1}{\mathrm{B}}}}{2}+\frac{2\ddot{e}\overset{e}{\mathrm{B}}+\dot{e}\overset{e}{\underset{1}{\mathrm{B}}}}{2}+\&c.$$

$$\mathrm{B}=\frac{3\dddot{c}\overset{c}{\mathrm{B}}+2\ddot{c}\overset{c}{\underset{1}{\mathrm{B}}}+\dot{c}\overset{c}{\underset{2}{\mathrm{B}}}}{3}+\frac{3\dddot{e}\overset{e}{\mathrm{B}}+2\ddot{e}\overset{e}{\underset{1}{\mathrm{B}}}+\dot{e}\overset{e}{\underset{2}{\mathrm{B}}}}{3}+\&c.$$

. . . . . .

$$\mathrm{B}=\frac{n\ldots n\dot{c}\overset{c}{\mathrm{B}}+(n-1)\dot{c}\overset{c}{\underset{1}{\mathrm{B}}}+\ldots\ldots+2\ddot{c}\overset{c}{\underset{n-2}{\mathrm{B}}}+\dot{c}\overset{c}{\underset{n-1}{\mathrm{B}}}}{n}$$

$$+\frac{n\ldots n\dot{e}\overset{e}{\mathrm{B}}+(n-1)\dot{e}\overset{e}{\underset{1}{\mathrm{B}}}+\ldots\ldots+2\ddot{e}\overset{e}{\underset{n-2}{\mathrm{B}}}+\dot{e}\overset{e}{\underset{n-1}{\mathrm{B}}}}{n}+\&c.\ldots\ldots(\delta).$$

But $\mathrm{B}=\phi\left\{\mathrm{F}(c),f(e),\&c.\right\}$ whence all the coefficients are known.

10. This solution, however, gives no very expeditious way of actually expanding the function in question; particularly when we get to the higher powers: but by proceeding as in the second method, made use of for a function of one single multinomial, we find

$$\mathrm{B}=\int\overset{c}{\underset{n-1}{\mathrm{B}}}\dot{c}+\int\overset{c}{\underset{n-2}{\mathrm{B}}}\ddot{c}+\int\overset{c}{\underset{n-3}{\mathrm{B}}}\dddot{c}+\ldots\ldots+\int\overset{e}{\underset{n-1}{\mathrm{B}}}\dot{e}+\int\overset{e}{\underset{n-2}{\mathrm{B}}}\ddot{e}+\int\overset{e}{\underset{n-3}{\mathrm{B}}}\dddot{e}+$$
$$\ldots\ldots+\&c.\ldots\ldots(\varepsilon)$$

where we must neglect in $\overset{c}{\underset{n-2}{\mathrm{B}}}$ all terms which contain $\dot{c}$; in $\overset{c}{\underset{n-3}{\mathrm{B}}}$

all those which contain $\dot{c}$ or $\ddot{c}$; and so on. In $\overset{\acute{e}}{B}_{n-1}$ must be neglected all terms which contain any of the c's; in $\overset{\acute{e}}{B}_{n-2}$ these, and those also containing $\dot{e}$; and, *in general, all those terms must be neglected, as we proceed, which contain any quantities whose fluxions have entered into the preceding terms.*

From the above equation is derived an easy mode of expansion, I shall give an example in the case of two functions; and shall represent, for brevity, $\varphi\left\{F(c), f(e)\right\}$ by φ, and its fluxional coefficient of the $\overline{m+n}$th order (when the fluxion has been taken m times with respect to c and n times with respect to e) by $\overset{m,n}{\varphi}$.

We find here, $B = \varphi$; $\underset{1}{B} = \overset{1,0}{\varphi}\dot{c} + \overset{0,1}{\varphi}\dot{e}$;

$\underset{2}{B} = \overset{1,0}{\varphi}\ddot{c} + \overset{2,0}{\varphi}\frac{\dot{c}^2}{2} + \overset{0,1}{\varphi}\ddot{e} + \overset{0,2}{\varphi}\frac{\dot{e}^2}{2} + \overset{1,1}{\varphi}\dot{c}\dot{e}$; but to explain more fully the manner of proceeding, let it be required to find $\underset{3}{B}$ from the preceding coefficients. We have, after neglecting the specified terms,

$\int \underset{2}{\dot{B}}\dot{c} = \overset{2,0}{\varphi}\dot{c}\dot{c} + \overset{3,0}{\varphi}\frac{\dot{c}^3}{2.3} + \overset{1,1}{\varphi}\dot{c}\dot{e} + \overset{1,2}{\varphi}\dot{c}\frac{\dot{e}^2}{2} + \overset{2,1}{\varphi}\frac{\dot{c}^2}{2}\dot{e}$; $\int \underset{1}{\dot{B}}\dot{c} = \overset{1,1}{\varphi}\dot{c}\dot{e}$

$\int \underset{2}{\dot{B}}\dot{c} = \overset{1,0}{\varphi}\ddot{e}$; $\int \underset{2}{\dot{B}}\dot{e} = \overset{0,2}{\varphi}\dot{e}\dot{e} + \overset{0,3}{\varphi}\frac{\dot{e}^3}{2.3}$; $\int \underset{1}{\dot{B}}\dot{e} = \overset{0,1}{\varphi}\dot{e}$; the sum of these gives

$\underset{3}{B} = \overset{1,0}{\varphi}\ddot{c} + \overset{2,0}{\varphi}\dot{c}\dot{c} + \overset{3,0}{\varphi}\frac{\dot{c}^3}{2.3} + \overset{0,1}{\varphi}\ddot{e} + \overset{0,2}{\varphi}\dot{e}\dot{e} + \overset{0,3}{\varphi}\frac{\dot{e}^3}{2.3} + \overset{1,1}{\varphi}(\dot{c}\ddot{e} + \ddot{c}\dot{e})$

$+ \overset{1,2}{\varphi}\dot{c}\frac{\dot{e}^2}{2} + \overset{2,1}{\varphi}\frac{\dot{c}^2}{2}\dot{e}$.

11. To get methods of deriving any coefficient from the one immediately preceding it, we must substitute, in (δ) and (ϵ), the values of $\overset{c}{\underset{n-m}{\mathrm{B}}}$ and $\overset{e}{\underset{n-m}{\mathrm{B}}}$ given by the following equations, which are similar to those found before, in the case of one multinomial, and marked (4) and (5).

$$\overset{c}{\underset{n-m}{\mathrm{B}}} = \left(\frac{\dot{\mathrm{B}}}{\overset{n}{\underset{\cdots m}{c}}}\right) = \left(\frac{\dot{\mathrm{B}}}{\overset{n-1}{\underset{\cdots(m-1)}{c}}}\right); \quad \overset{e}{\underset{n-m}{\mathrm{B}}} = \left(\frac{\dot{\mathrm{B}}}{\overset{n}{\underset{\cdots m}{e}}}\right) = \left(\frac{\dot{\mathrm{B}}}{\overset{n-1}{\underset{\cdots(m-1)}{e}}}\right);$$

&c. ..... (η)

$$\overset{c}{\underset{n-m}{\mathrm{B}}} = \left(\frac{\overset{m-1}{\dot{\mathrm{B}}}}{\overset{n-1}{\underset{,m-1}{c}}}\right)_{cc\ldots(m-2)}; \quad \overset{e}{\underset{n-m}{\mathrm{B}}} = \left(\frac{\overset{m-1}{\dot{\mathrm{B}}}}{\overset{n-1}{\underset{,m-1}{e}}}\right)_{ee\ldots(m-2)}; \text{ &c. ...} (\theta)$$

where, by any number of c's or e's placed under a quantity, I represent the depression of the fluxional coefficients of φ, contained in that quantity, so many orders, with respect to c or e.

By combining equations (η) and (δ), we find

$$n\,\underset{n}{\mathrm{B}} = n\,\overset{n\ldots n}{c}\left(\frac{\dot{\mathrm{B}}}{\overset{n-1}{\underset{\cdots(n-1)}{c}}}\right) + (n-1)\,\overset{n\ldots(n-1)}{c}\left(\frac{\dot{\mathrm{B}}}{\overset{n-1}{\underset{\cdots(n-2)}{c}}}\right) + \ldots\ldots$$

$$+ 2\,\overset{n}{c}\left(\frac{\dot{\mathrm{B}}}{\overset{n-1}{\underset{,}{e}}}\right) + \overset{'}{c}\left(\frac{\dot{\mathrm{B}}}{\overset{n-1}{\underset{,}{c}}}\right) + n\,\overset{n\ldots n}{e}\left(\frac{\dot{\mathrm{B}}}{\overset{n-1}{\underset{\cdots(n-1)}{e}}}\right) + (n-1)\,\overset{n\ldots(n-1)}{e}$$

$$\left(\frac{\dot{\mathrm{B}}}{\overset{n-1}{\underset{\cdots(n-2)}{e}}}\right) + \ldots. + 2\,\overset{n}{e}\left(\frac{\dot{\mathrm{B}}}{\overset{n-1}{\underset{,}{e}}}\right) + \overset{'}{e}\left(\frac{\dot{\mathrm{B}}}{\overset{n-1}{\underset{,}{e}}}\right) + \text{ &c. which in}$$

words is this: *To find* $\underset{n}{\mathrm{B}}$, *take the fluxion of* $\underset{n-1}{\mathrm{B}}$ *with respect to*

$\dot{c}, \dot{\dot{c}}, \dot{\dot{\dot{c}}}$, &c. $e, \dot{e}, \dot{\dot{e}}, \dot{\dot{\dot{e}}}$, &c. *and change, every where,* $c^{\cdots m}$ *into* $c^{\cdots(m+1)} \times \frac{m+1}{n}$, *and* $e^{\cdots m}$ *into* $e^{\cdots(m+1)} \times \frac{m+1}{n}$, &c.

The reader may try this rule on the examples in the last article: I proceed to simpler methods for practice.

By combining equations (θ) and (ι), and considering $\overset{}{\mathrm{B}}_{n-1}$ under the forms $\mathrm{B}_{n-1} = \beta + \overset{(1)}{\beta}\dot{c} + \overset{(2)}{\beta}\dot{\dot{c}} + \overset{(3)}{\beta}\dot{\dot{\dot{c}}} + \ldots$, and $\mathrm{B}_{n-1} = \Delta + \overset{(1)}{\Delta}\dot{e} + \overset{(2)}{\Delta}\dot{\dot{e}} + \overset{(3)}{\Delta}\dot{\dot{\dot{e}}} + \ldots$, and &c. we find $\mathrm{B}_n = \int \mathrm{B}_{n-1}\dot{c} + \int \overset{(1)}{\beta}\dot{c} + \int \overset{(2)}{\beta}\dot{c} + 2\int \overset{(2)}{\beta}\dot{c} + 2.3\int \overset{(3)}{\beta}\dot{c} + \ldots\ldots + \int \mathrm{B}_{n-1}\dot{e} + \int \overset{(1)}{\Delta}\dot{e} + 2\int \overset{(2)}{\Delta}\dot{e} + 2.3\int \overset{(3)}{\Delta}\dot{e} + \ldots\ldots +$ &c. where in $\overset{(2)}{\beta}$ all terms must be neglected which contain $\ddot{c}$; and in general, according the rule given in article 10, all terms must be neglected that contain any quantities whose fluxions have entered into the preceding terms.

By this method the expansion might be accomplished without difficulty, each term is found at once, and no reductions are necessary: the one which I am going to give is, however, much better, being, I conceive, the simplest possible.

12. By combining the equations (η) and (ι), there results

$$\mathrm{B}_n = \int \left(\frac{\dot{\mathrm{B}}_{n-1}}{\dot{c}}\right)\dot{c} + \int \left(\frac{\dot{\mathrm{B}}_{n-1}}{\dot{c}}\right)\dot{c} + \int \left(\frac{\dot{\mathrm{B}}_{n-1}}{\ddot{c}}\right)\dot{c} + \ldots\ldots$$
$$+ \int \left(\frac{\dot{\mathrm{B}}_{n-1}}{\dot{e}}\right)\dot{e} + \int \left(\frac{\dot{\mathrm{B}}_{n-1}}{\dot{c}}\right)\dot{e} + \int \left(\frac{\dot{\mathrm{B}}_{n-1}}{\ddot{c}}\right)\dot{e} + \ldots +, \text{&c.}$$

where we must observe to neglect certain terms, according to the directions so often given: and if we apply here all that was said in article 6, when we were considering a similar expression for a function of one multinomial, we easily get the following

Rule.

To find $B^{(n)}$ *from* $B^{(n-1)}$, *in the expansion of a function of any functions of the multinomials,* $c + \dot{c}x + \ddot{c}x^2 +$ *and* $e + \dot{e}x + \ddot{e}x^2 +$ *and* $d + \dot{d}x + \ddot{d}x^2 +$ *and &c.*

1*st. Consider only the c's, and take the fluxion of* $B^{(n-1)}$, *with respect to the last of them in each term; and the last but one also, if it immediately precede the last in the number of its strokes: change, every where,* $c^{\cdot\cdots m}$ *into* $c^{\cdot\cdots(m+1)}$, *and take the fluent of each term with respect to this last.*

2*dly. Neglect all terms in* $B^{(n-1)}$ *which contain* $\dot{c}, \ddot{c}, \dddot{c}$, *&c. and proceed, with the remaining ones, in the same manner with respect to the e's.*

3*dly. Neglect all terms in* $B^{(n-1)}$ *which contain* $\dot{c}, \ddot{c}, \dddot{c}$, *&c.* $\dot{e}, \ddot{e}, \dddot{e}$, *&c. and proceed, with the remaining ones, in the same manner with respect to the d's.—And so on.*

Let it be required, in the case of two multinomials, to find $B^{(4)}$ from $B^{(3)}$ which is given in article 10. The first part of the rule gives

$$\varphi^{1,0}_{m}\, c + \varphi^{2,0}_{1\,m}\left(\dot{c}c + \frac{\ddot{c}^2}{2}\right) + \varphi^{3,0}_{\dot{c}^2\,\ddot{}}\,\frac{\dot{c}^2}{2}\,\dddot{c} + \varphi^{4,0}_{\dddot{c}^4}\,\frac{\dddot{c}^4}{2\cdot 3\cdot 4} + \varphi^{1,1}_{\ddot{}\ \ddot{}}\,\dddot{c}\,\dddot{e} + \varphi^{2,1}_{\dot{c}^2\ \ddot{}}\,\frac{\dot{c}^2}{2}\,\dddot{e} + \varphi^{1,1}_{\dddot{}\ 1}\,\dddot{c}\,e$$

$$+\varphi^{1,2}\,{}^{''}c\frac{{}^{'}e^{2}}{2}+\varphi^{2,2}\,\frac{{}^{'}c^{2}}{2}\frac{{}^{''}e^{2}}{2}+\varphi^{2,1}\,{}^{'}c\,{}^{''}c\,{}^{''}e+\varphi^{3,1}\,\frac{{}^{'}c^{3}}{2.3}\,{}^{''}e+\varphi^{1,1}\,{}^{'}c\,{}^{'''}e+\varphi^{1,2}\,{}^{'}c\,{}^{''}e\,{}^{''}e+\varphi^{1,3}\,{}^{'}c\frac{{}^{''}e^{3}}{2.3}.$$

The second part gives

$$\varphi^{0,1}\,{}^{'''}e+\varphi^{0,2}\left({}^{'}e\,{}^{'''}e+\frac{{}^{''}e^{2}}{2}\right)+\varphi^{0,3}\frac{{}^{'}e^{3}}{2}\,{}^{''}e+\varphi^{0,4}\,\frac{{}^{'}e^{4}}{2.3.4}.$$

The sum of these is $\overset{4}{\mathrm{B}}$. As the number of multinomials adds nothing to the difficulty of expansion, according to this method, it is useless to give more examples.

19. Nor does the number of multinomials make any difference as to the facility of *inverse derivation;* which depends on the equation

$$\underset{n-m}{\mathrm{B}}=\left(\begin{matrix}\overset{\cdot}{\mathrm{B}}\\ \frac{n}{n\ldots m}\\ c\ \cdot\end{matrix}\right)_{e}$$

Thus from $\overset{4}{\mathrm{B}}$, just now given, in the case of two multinomials, let it be required to find $\overset{3}{\mathrm{B}}$; we have

$$\underset{2}{\mathrm{B}}=\underset{4-2}{\mathrm{B}}=\left(\begin{matrix}\overset{\cdot}{\mathrm{B}}\\ \frac{4}{n}\\ c\cdot\end{matrix}\right)_{e}=\varphi^{1,0}\,{}^{''}c+\varphi^{2,0}\,\frac{{}^{'}c^{2}}{2}+\varphi^{0,1}\,{}^{''}e+\varphi^{0,2}\,\frac{{}^{'}e^{2}}{2}+\varphi^{1,1}\,{}^{'}c\,{}^{''}e.$$

14. There remains the important

Problem.

To find $\overset{r}{\mathrm{B}}$ *without knowing any of the other coefficients.*

It will be plain to any one, who in the least considers the methods that have been employed, that $\overset{r}{\mathrm{B}}$ must contain all the possible combinations of ${}^{'}c, {}^{''}c, {}^{'''}c$, &c. ${}^{'}e, {}^{''}e, {}^{'''}e$, &c. ${}^{'}d, {}^{''}d, {}^{'''}d$, &c. &c. that can be formed with this condition, that the number of strokes be r. Every mth power will be divided by the product

$2.3.4....m$: and the fluxional coefficient $^{\alpha,\beta,\gamma,\&c.}_{\varphi}$, that multiplies any term, will have for the left hand figure over it that number which is the sum of the exponents of the c's; for the next figure on the same side that number which is the sum of the exponents of the e's; for the third that which is the sum of the exponents of the d's; and so on.

The only difficulty then is to find these combinations (without the possibility of missing any, or the trouble of finding the same more than once) by some regular process of derivation.

A rule was given in Art. 8, when we were considering the similar problem in the case of one multinomial, for deriving all the combinations in B, in which the sum of the strokes is r, from $\overset{r}{c}$ as origin of derivation.

The same rule will apply here, but instead of the one origin $\overset{r}{c}$, we have, in the case of two multinomials, the origins

$$\overset{r}{c}, \overset{r-1}{c}\overset{1}{e}, \overset{r-2}{c}\overset{2}{e}, \ldots\ldots \overset{2}{c}\overset{r-2}{e}, \overset{1}{c}\overset{r-1}{e}, \overset{r}{e}.$$

Let us consider any particular origin as $\overset{n}{c}\overset{m}{e}$. I denote the term derived immediately from $\overset{n}{c}$ (by the rule in Art. 8,) by $\Delta\overset{n}{c}$; and the terms derived from this last, from the same rule by $\Delta^2\overset{n}{c}$; those got from $\Delta^2\overset{n}{c}$ by $\Delta^3\overset{n}{c}$; and so on.

It is evident that all the possible combinations (of the kind

* $^{\alpha,\beta,\gamma,\&c.}_{\varphi}$ represents the fluxional coefficient of $\varphi\left\{F(c), f(e), V(d), \&c.\right\}$ of the order $\alpha+\beta+\gamma+$, &c. where the fluxion has been taken α times with respect to c, β times with respect to e, and γ times with respect to d; and so on.

we are seeking) derived from the origin $\dot{c}^n \dot{e}^m$, will be expressed by the product

$$(\dot{c}^n + \Delta \dot{c}^n + \Delta^2 \dot{c}^n + \Delta^3 \dot{c}^n + \&c.)\,(\dot{e}^m + \Delta \dot{e}^m + \Delta^2 \dot{e}^m + \Delta^3 \dot{e}^m + \&c.),$$

where each derivation of $\dot{e}^m$ is multiplied by every one of $\dot{c}^n$, and conversely each one of $\dot{c}^n$ by every one of $\dot{e}^m$.

We have nothing to do then but to deduce the derivations from $\dot{c}^n$ and $\dot{e}^m$ by the rule in Art. 8.

Suppose that B was the coefficient required, and that we wanted all the combinations arising from $\overset{5}{c}{}^3 \dot{e}^2$. We have here

$$\Delta \dot{c}^3 = \dot{c}\,\ddot{c}; \quad \Delta^2 \dot{c}^3 = \dddot{c}; \quad \Delta^3 \dot{c}^3 = 0; \quad \Delta \dot{e}^2 = \ddot{e}; \quad \Delta^2 \dot{e}^2 = 0,$$

and, by substituting these values in the above product, we find all the combinations arising from the origin $\dot{c}^3 \dot{e}^2$ to be $\dot{c}^3 \dot{e}^2 + \dot{c}^3 \ddot{e} + \dot{c}\,\ddot{c}\,\dot{e}^2 + \dddot{c}\,\dot{e}^2 + \dot{c}\,\ddot{c}\,\ddot{e} + \dddot{c}\,\ddot{e}$.

It would be as well to write down the appropriate denominators to each combination as we proceed: and when we had treated all the origins of derivation in this manner, there would only remain to arrange the terms under their proper fluxional coefficients.*

* Instead of $\dot{c}^n \dot{e}^m$, I might have taken for origin of derivation $\overset{n,0}{\varphi}\, \dot{c}^n \times \overset{0,m}{\varphi}\, \dot{e}^m$; and after multiplying the factors

$$(\overset{n,0}{\varphi}\, \dot{c}^n + \overset{n-1,0}{\varphi}\, .\Delta \dot{c}^n + \overset{n-2,0}{\varphi}\, .\Delta^2 \dot{c}^n + \&c.)\,(\overset{0,m}{\varphi}\, \dot{e}^m + \overset{0,m-1}{\varphi}\, .\Delta \dot{e}^m + \overset{0,m-2}{\varphi}\, .\Delta^2 \dot{e}^m + \&c.)$$

have changed $\overset{\nu,0}{\varphi} \times \overset{0,\mu}{\varphi}$ into $\overset{\nu,\mu}{\varphi}$; but this would only give additional trouble without answering any useful end: it is sufficiently plain that the appropriate fluxional coefficient of $\Delta^\nu \dot{c}^n \times \Delta^\mu \dot{e}^m$ will be $\overset{n-\nu,m-\mu}{\varphi}$.

If the function under consideration contain three multinomials, the origins of derivation will be

$$c^r \quad , c^{r-1}e, c^{r-2}e^2, \ldots\ldots\ldots c^2e^{r-2}, ce^{r-1}, e^r$$

$$d \times (c^{r-1}, c^{r-2}e, c^{r-3}e^2, \ldots\ldots\ldots ce^{r-2}, e^{r-1})$$

$$d^2 \times (c^{r-2}, c^{r-3}e, c^{r-4}e^2, \ldots\ldots\ldots e^{r-2})$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$d^{r-2} \times (c^2 \quad , ce \quad , e^2)$$

$$d^{r-1} \times (c \quad , e)$$

$$d^r$$

and all the possible combinations derived from each particular origin as, $c^n e^m d^p$, will be expressed by the product

$$(c^n + \Delta c^n + \Delta^2 c^n +, \text{ \&c.}) \quad (e^m + \Delta e^m + \Delta^2 e^m +, \text{ \&c.})$$

$$(d^p + \Delta d^p + \Delta^2 d^p +, \text{ \&c.})$$

The reader will easily extend the method, if necessary, to a greater number of multinomials.

As we have, in this manner, a certain, easy, and regular way of finding all the combinations in any coefficient B_r, the problem is completely solved.

I go on to *multinomials of higher kinds:* and, with M. ARBOGAST, shall call those multinomials of the nth order which are disposed according to the powers and products of n different letters x, y, z, &c.

15. After having so fully entered into particulars, in the preceding cases, there can be no difficulty in perceiving, that *a complete theory of derivation, for the expansion of any function of any number of functions of multinomials, whether they be simple,*

or double, or triple, &c. is contained in the following equations.

$$\underset{m,\,n,\,r,\,\&c.}{B} = \Sigma.\int \underset{m-\mu,\,n-\nu,\,r-\rho,\,\&c.}{\overset{c}{B}} \times \underset{\mu,\,\nu,\,\rho,\,\&c.}{\overset{c}{\mu}} + \Sigma.\int \underset{m-\mu,\,n-\nu,\,r-\rho,\,\&c.}{\overset{d}{B}} \times \underset{\mu,\,\nu,\,\rho,\,\&c.}{\overset{d}{\mu}} + \&c. \ldots\ldots (\varkappa)$$

$$\underset{m-\mu,\,n-\nu,\,r-\rho,\,\&c.}{B} = \left(\frac{\overset{A}{B}_{m,n,r,\&c.}}{\underset{\mu,\,\nu,\,\rho,\,\&c.}{\overset{c}{\mu}}}\right)_{c} = \&c. \ldots\ldots (\lambda),$$

where $\underset{m,\,n,\,r,\,\&c.}{B}$ is the coefficient of $x^m y^n z^r$, &c. in the expansion; $\underset{\mu,\,\nu,\,\rho,\,\&c.}{\overset{c}{\mu}}$ &c. and $\underset{\mu,\,\nu,\,\rho,\,\&c.}{\overset{d}{\mu}}$ &c. are the coefficients of $x^{\mu} y^{\nu} z^{\rho}$ &c. before expansion, under the signs of the functions.

The sign Σ, in the first formula, expresses the sum of all the terms that can be formed by taking for μ, ν, ρ, &c. all the whole numbers from o to m, n, r, &c. μ, ν, ρ, &c. must not however all equal nothing at the same time.

It is scarcely necessary to observe, that certain terms are understood to be neglected in equation $(\varkappa)$, according to the rule given in article 10, which is, that all terms in the B's must be neglected, as we proceed, which contain quantities whose fluxions enter into the preceding terms.

The above expressions if considered not only in themselves, but with respect to the formulas that are immediately deducible from their developement and combination, in the manner that will presently be shewn, appear to be the most general and important in this branch of analysis.

16. Let it be required to expand the double multinomial function

$$\varphi\left\{\begin{array}{l} c + c\,x + c\,x^2 + c\,x^3 + \\ \quad + c\,y + c\,x\,y + c\,x^2\,y + \\ \qquad + c\,y^2 + c\,x\,y^2 + \\ \qquad\quad + c\,y^3 + \end{array}\right\} \text{into the form} \quad \begin{array}{l} B + B\,x + B\,x^2 + B\,x^3 + \\ \quad + B\,y + B\,x\,y + B\,x^2\,y + \\ \qquad + B\,y^2 + B\,x\,y^2 + \\ \qquad\quad + B\,y^3 + \end{array}$$

Equation (x) is in this case reduced to $B = \Sigma . \int^{\prime}_{m-\mu,n-\nu} B \times \mu,\nu$ which expanded is

$$(\mu) \ldots\ldots B = \int^{\prime}_{m-1,n} B \; {}^{c}_{1,0} + \int^{\prime}_{m-2,n} B \; {}^{c}_{2,0} + \ldots\ldots\ldots\ldots$$
$$+ \int^{\prime}_{m,n-1} B \; {}^{c}_{0,1} + \int^{\prime}_{m-1,n-1} B \; {}^{c}_{1,1} + \ldots\ldots\ldots\ldots$$
$$+ \int^{\prime}_{m,n-2} B \; {}^{c}_{0,2} + \ldots\ldots\ldots\ldots$$
$$+$$

By this formula, we find the first terms of the expansion to be

$$\varphi(c) + \varphi'(c)\,{}^{c}_{1,0}\,x + \varphi'(c)\,{}^{c}_{2,0}\,\Big|\,x^2 +$$
$$+ \varphi'(c)\,{}^{c}_{0,1}\,y + \varphi''(c)\,\left({}^{c}_{\frac{1,0}{2}}\right)^2\Big|$$
$$+ \varphi'(c)\,{}^{c}_{1,1}\,\Big|\,x\,y +$$
$$+ \varphi''(c)\,{}^{c}_{1,0} \times {}^{c}_{0,1}\Big|$$
$$+$$

Let it be required to find $B_{2,1}$. We have, after neglecting the terms specified,

$\int \overset{\prime}{B}_{1,1} \overset{c}{}_{1,0} = \overset{''}{\varphi}(c) \overset{c}{}_{1,0} \times \overset{c}{}_{1,1} + \overset{'''}{\varphi}(c) \left(\frac{\overset{c}{}_{1,0}}{2}\right)^{2} \times \overset{c}{}_{0,1}$; $\int \overset{\prime}{B}_{2,0} \overset{c}{}_{0,1} = \overset{'}{\varphi}(c) \overset{c}{}_{0,1} \times \overset{c}{}_{2,0}$;

$\int \overset{\prime}{B}_{2,1} \overset{c}{}_{2,1} = \overset{'}{\varphi}(c) \overset{c}{}_{2,1}$; and these added together give

$$\overset{\prime}{B}_{2,1} = \overset{'}{\varphi}(c) \overset{c}{}_{2,1} + \overset{''}{\varphi}(c) \left\{ \overset{c}{}_{1,0} \times \overset{c}{}_{1,1} + \overset{c}{}_{0,1} \times \overset{c}{}_{2,0} \right\} + \overset{'''}{\varphi}(c) \left(\frac{\overset{c}{}_{1,0}}{2}\right)^{2} \times \overset{c}{}_{0,1}.$$

I wrote the terms separately, and then collected them, for the better explanation of the method; but this double labour is by no means necessary: the coefficients may be formed and written down at once, as quickly as can be wished.

17. A very little consideration will convince us, that the terms $\int \overset{\prime}{B}_{m,n-1} \overset{c}{}_{0,1}$; $\int \overset{\prime}{B}_{m,n-2} \overset{c}{}_{0,2}$; ..... $\int \overset{\prime}{B}_{m,n-r} \overset{c}{}_{0,r}$ may be entirely left out of formula (μ), excepting when the term we search is of the form $B_{0,m}$, in which case it is reduced to the coefficient of y^m in the expansion of $\varphi(c + \overset{c}{}_{0,1} y + \overset{c}{}_{0,2} y^2 + \ldots)$.

If then we neglect these terms, and, in the remaining ones, put for $\overset{\prime}{B}_{m-\mu,n-\nu}$ its value $\left(\frac{\overset{\prime}{B}_{m-1,n}}{\overset{c}{}}\right)_{\mu-1,\nu}$ given by (λ), equation (μ) will become

$$(\nu) \ldots\ldots B_{m,n} = \int \left(\frac{\overset{\prime}{B}_{m-1,n}}{\overset{c}{}}\right) \overset{c}{}_{1,0} + \int \left(\frac{\overset{\prime}{B}_{m-1,n}}{\overset{c}{}}\right)_{1,0} \overset{c}{}_{2,0} + \ldots\ldots\ldots\ldots$$

$$+ \int \left(\frac{\overset{\prime}{B}_{m-1,n}}{\overset{c}{}}\right)_{0,1} \overset{c}{}_{1,1} + \ldots\ldots\ldots,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots + \int \left(\frac{\overset{\prime}{B}_{m-1,n}}{\overset{c}{}}\right)_{\mu-1,\nu} \overset{c}{}_{\mu,\nu} + \ldots.$$

by means of which equation we may find $\dot{B}_{m,n}$ from $\dot{B}_{m-1,n}$. Here, as usual, we must omit, in the successive fluxional coefficients, all terms containing quantities whose fluxions have been factors in the preceding terms of the formula. Let us, for an example, find $\dot{B}_{3,1}$ from $\dot{B}_{2,1}$ given in the last article; we have

$$\int\left(\frac{\dot{B}_{2,1}}{\dot{c}}\right)\dot{c}_{1,0}=\ddot{\phi}(c)\,c_{1,0}\times c_{2,1}+\dddot{\phi}(c)\left\{\frac{\left(c_{1,0}\right)^2}{2}\times c_{1,1}+c_{1,0}\times c_{0,1}\times c_{2,0}\right\}+\dddot{\phi}(c)\frac{\left(c_{1,0}\right)^3}{2.3}\times c_{0,1};\quad \int\left(\frac{\dot{B}_{2,1}}{\dot{c}}\right)\dot{c}_{2,0}=\ddot{\phi}c_{1,1}\times c_{2,0};\quad \int\left(\frac{\dot{B}_{2,1}}{\dot{c}}\right)\dot{c}_{3,0}=\ddot{\phi}(c)\,c_{0,1}\times c_{3,0};\quad \int\left(\frac{\dot{B}_{2,1}}{\dot{c}}\right)\dot{c}_{2,1}=\dot{\phi}(c)\,c_{3,1};$$

whence $\dot{B}_{3,1}=\dot{\phi}(c)\,c_{3,1}+\ddot{\phi}(c)\left\{c_{1,0}\times c_{2,1}+c_{1,1}\times c_{3,0}+c_{0,1}\times c_{3,0}\right\}+\dddot{\phi}(c)\left\{\frac{\left(c_{1,0}\right)^2}{2}\times c_{1,1}+c_{1,0}\times c_{0,1}\times c_{2,0}\right\}+\dddot{\phi}(c)\frac{\left(c_{1,0}\right)^2}{2.3}\times c_{0,1}.$

18. We may also derive from equation (r) the following simple

Rule.

To find $\dot{B}_{m,n}$, *take the fluxion of* $\dot{B}_{m-1,n}$ *with respect to all the quantities; change, every where,* $\dot{c}_{\mu,\nu}$ *into* $\dot{c}_{\mu+1,\nu}$, *and take the fluent with respect to this last. The same terms must be kept only once.* c *is* $c_{0,0}$.

By this rule we frequently find the same terms more than once, which disadvantage is, however, more than compensated by its shortness, and the ease and simplicity of the process.

Let it be required from $B_{0,3}$ to find successively $B_{1,3}$, $B_{2,3}$, &c.

We saw, when treating of a simple multinomial, that

$$B_{0,3} = \phi'(c)\,c_{0,3} + \phi''(c)\,c_{0,1}\times c_{0,2} + \phi'''(c)\left(\frac{c_{0,1}}{2\cdot 3}\right)^3$$ whence, by the rule,

$$B_{1,3} = \phi'(c)\,c_{1,3} + \phi''(c)\left\{c_{1,0}\times c_{0,3} + c_{1,1}\times c_{0,2} + c_{0,1}\times c_{1,2}\right\} + \phi'''(c)$$

$$\left\{c_{1,0}\times c_{0,1}\times c_{0,2} + \left(\frac{c_{0,1}}{2}\right)^2\times c_{1,1}\right\} + \phi''''(c)\,c_{1,0}\times\left(\frac{c_{0,1}}{2\cdot 3}\right)^3;\quad B_{2,3} = \phi'(c)\,c_{2,3}$$

$$+\phi''(c)\left\{c_{2,0}\times c_{0,3} + c_{1,0}\times c_{1,3} + c_{2,1}\times c_{0,2} + c_{1,1}\times c_{1,2} + c_{0,1}\times c_{2,2}\right\}$$

$$+\phi'''(c)\left\{\left(\frac{c_{1,0}}{2}\right)^2\times c_{0,3} + c_{1,0}\times c_{1,1}\times c_{0,2} + c_{1,0}\times c_{0,1}\times c_{1,2} + c_{2,0}\times c_{0,1}\right.$$

$$\left.\times c_{0,2} + c_{0,1}\left(\frac{c_{1,1}}{2}\right)^2 + \left(\frac{c_{0,1}}{2}\right)^2\times c_{2,1}\right\} + \phi''''(c)\left\{\left(\frac{c_{1,0}}{2}\right)^2\times c_{0,1}\times c_{0,2} + c_{1,0}\right.$$

$$\left.\times\left(\frac{c_{0,1}}{2}\right)^2\times c_{1,1} + c_{2,0}\times\left(\frac{c_{0,1}}{2\cdot 3}\right)^3\right\} + \phi'''''(c)\left(\frac{c_{1,0}}{2}\right)^2\times\left(\frac{c_{0,1}}{2\cdot 3}\right)^3.$$ It is not necessary to calculate all the coefficients we may want by *direct derivation;* when we have got a few, in this manner, we may find the rest by the *inverse method* which is much easier. M. Arbogast has put the twenty-eight first terms in a table;* of these there was need to calculate *only four directly*, as I shall show hereafter. But, to give an example of this inverse proceeding, let it be required to find $B_{2,2}$ from $B_{2,3}$ just now given.

Equation (λ) becomes in the present case $B_{m-\mu,\,n-\nu} = \left(\frac{\dot{B}}{\frac{m,n}{c}}\right)_{\mu,\nu}$;

whence $B_{2,2} = B_{2-0,\,3-1} = \left(\frac{\dot{B}}{\frac{2,3}{c}}\right)_{0,1} = \phi'(c)\,c_{2,2} + \phi''(c)\left\{c_{1,0}\times c_{1,2}\right.$

* *Calc. des Deriv.* p. 127.

$$+ \overset{e}{}_{2,0} \times \overset{c}{}_{0,2} + \frac{\left(\overset{c}{}_{1,1}\right)^2}{2} + \overset{c}{}_{0,1} \times \overset{e}{}_{2,1} \Big\} + \overset{m}{\varphi}(c) \Big\{ \frac{\left(\overset{e}{}_{1,0}\right)^2}{2} \times \overset{e}{}_{0,2} + \overset{e}{}_{1,0} \times \overset{e}{}_{0,1}$$

$$\times \overset{c}{}_{1,1} + \overset{c}{}_{2,0} \times \frac{\left(\overset{e}{}_{0,1}\right)^2}{2} \Big\} + \overset{m}{\varphi}(c) \frac{\left(\overset{e}{}_{1,0}\right)^2}{2} \times \frac{\left(\overset{c}{}_{0,1}\right)^2}{2}.$$

19. Instead of leaving out of equation (μ) the terms of this form $\int \overset{\dot{B}}{}_{m,n-r} \overset{e}{}_{0,r}$ as we did in article 17, we might have omitted those of the form $\int \overset{\dot{B}}{}_{m-r,n} \overset{e}{}_{r,0}$; in which case it would have become

$$\mathrm{B} = \int \overset{\dot{B}}{}_{m,n-1} \overset{e}{}_{0,1} + \int \overset{\dot{B}}{}_{m-1,n-1} \overset{e}{}_{1,1} + \ldots\ldots$$

$$\int \overset{\dot{B}}{}_{m,n-2} \overset{e}{}_{0,2} + \ldots\ldots$$

$$\ldots\ldots$$

$$\ldots\ldots + \int \overset{\dot{B}}{}_{m-\mu,n-\nu} \overset{e}{}_{\mu,\nu} + \ldots$$

Here, if we put for $\overset{\dot{B}}{}_{m-\mu,n-\nu}$ its value $\left(\frac{\overset{\dot{B}}{}_{m+1,n-1}}{\overset{c}{}_{\mu+1,\nu-1}}\right)$ derived from equation (λ), there results

$$\mathrm{B} = \int \left(\frac{\overset{\dot{B}}{}_{m+1,n-1}}{\overset{c}{}_{1,0}}\right) \overset{e}{}_{0,1} + \int \left(\frac{\overset{\dot{B}}{}_{m+1,n-1}}{\overset{c}{}_{1,0}}\right) \overset{e}{}_{1,1} + \ldots\ldots$$

$$+ \int \left(\frac{\overset{\dot{B}}{}_{m+1,n-1}}{\overset{c}{}_{1,1}}\right) \overset{e}{}_{0,2} + \ldots\ldots$$

$$\ldots\ldots + \int \left(\frac{\overset{\dot{B}}{}_{m+1,n-1}}{\overset{c}{}_{\mu+1,\nu-1}}\right) \overset{e}{}_{\mu,\nu} + \ldots\ldots$$

where, in every successive fluxional coefficient, certain terms are to be omitted, according to the usual rule.

Perhaps the simplest way of using this equation, although we shall frequently get the same combinations more than once, is by the following

Rule.

To find $\underset{m,n}{\mathrm{B}}$ *take the fluxion of* $\underset{m+1,\,n-1}{\mathrm{B}}$ *with respect to all the quantities, excepting* c, $\underset{0,1}{c}$, $\underset{0,2}{c}$, *&c.; change every where,* $\underset{\mu,\nu}{\dot{c}}$ *into* $\underset{\mu-1,\,\nu+1}{\dot{c}}$ *and take the fluent with respect to this last. The same terms must be kept only once.*

B was found in article 17, from which we have, by this rule;

$$\underset{2,2}{\mathrm{B}} = \overset{\prime}{\varphi}(c)\,\underset{2,2}{\dot{c}} + \overset{\prime\prime}{\varphi}(c)\left\{\underset{0,1}{c}\times\underset{2,1}{\dot{c}} + \underset{1,0}{\dot{c}}\times\underset{1,2}{\dot{c}} + \underset{0,2}{\dot{c}}\times\underset{2,0}{c} + \frac{\left(\underset{1,1}{c}\right)^{\cdot}}{2}\right\}$$

$$+\overset{\prime\prime\prime}{\varphi}(c)\left\{\frac{\left(\underset{1,0}{c}\right)^{\cdot}}{2}\times\underset{0,2}{\dot{c}} + \frac{\left(\underset{0,1}{c}\right)^{\cdot}}{2}\times\underset{2,0}{\dot{c}} + \underset{1,0}{\dot{c}}\times\underset{0,1}{\dot{c}}\times\underset{1,1}{c}\right\} + \overset{\mathrm{IV}}{\varphi}(c)\,\frac{\left(\underset{1,0}{c}\right)^{\cdot}}{2}$$

$$\times\frac{\left(\underset{0,1}{c}\right)^{\cdot}}{2}.$$

Suppose that, beginning with $\underset{6,0}{\mathrm{B}}$, we had calculated in this manner $\underset{5,1}{\mathrm{B}}$, $\underset{4,2}{\mathrm{B}}$, $\underset{3,3}{\mathrm{B}}$; from these may be found, with the greatest ease, *and without any more direct derivation*, the twenty-eight first terms. For from $\underset{5,1}{\mathrm{B}}$, $\underset{4,2}{\mathrm{B}}$ we find $\underset{1,5}{\mathrm{B}}$, $\underset{2,4}{\mathrm{B}}$ merely by changing, in the former, the numbers that are on the right hand of the commas (under the c's) to the left hand, and the reverse. All the other terms are found, by inverse derivation, from the equation $\underset{m-\mu,\,n-\nu}{\mathrm{B}} = \left(\frac{\underset{m,n}{\dot{\mathrm{B}}}}{\underset{\mu,\nu}{\dot{c}}}\right)_{1}$.

Problem.

20. *To find* $\mathrm{B}_{m,n}$ *immediately, without knowing any of the other coefficients.*

The coefficient of $x^m y^n$ will easily appear, from what has been shewn, to have the following form;

$$\mathrm{B}_{m,n} = \phi'(c)\,{}^{c}_{m,n} + \phi''(c)\,{}^{''}\!\downarrow + \ldots\ldots + \phi^{''\ldots(r-1)}(c)\,{}^{''\ldots(r-1)}\!\downarrow + \phi^{''\ldots r}(c)\,{}^{''\ldots r}\!\downarrow + \ldots$$

$$+ \phi^{''\ldots(m+n)}(c) \times \frac{\left({}^{c}_{1,0}\right)^m}{2.3\ldots m} \times \frac{\left({}^{c}_{0,1}\right)^n}{2.3\ldots n}$$

where $^{''\ldots r}\!\downarrow$ contains all the combinations that can be formed of the c's (after c or ${}^{c}_{0,0}$) in which the sum of the bottom figures, on the left of the commas, is m; the sum of those on the right n: and the number of factors r. Moreover every power as the mth will be divided by $2.3\ldots m$.

And the reader, who considers how the similar problem was solved, in the case of a simple multinomial, will have no difficulty in perceiving the reason of the following very simple

Rule.

To find $^{''\ldots(r-1)}\!\downarrow$ *from* $^{''\ldots r}\!\downarrow$, 1*st. take the fluxional coefficient, of the latter, with respect to* ${}^{c}_{1,0}$; *and, of this fluxional coefficient, take the fluxion with respect to all the quantities; change generally* ${}^{\dot{c}}_{\mu,\nu}$ *into* ${}^{\dot{c}}_{\mu+1,\nu}$, *and take the fluent with respect to this last.*

2*dly. Any terms in* $^{''\ldots r}\!\downarrow$ *of the form* $\frac{\left({}^{c}_{1,0}\right)^p}{2.3\ldots p} \times \frac{\left({}^{c}_{0,1}\right)^q}{2.3\ldots q} \times \frac{\left({}^{c}_{0,2}\right)^t}{2.3\ldots t} \times \frac{\left({}^{c}_{0,u}\right)^v}{2.3\ldots v} \times$ &c. *where, except in* ${}^{c}_{1,0}$, *all the figures are on the right of the commas, will require, besides, the following process. Take the fluxional coefficient with respect to* ${}^{c}_{0,1}$ *and, of this fluxional coefficient, take the*

fluxion with respect to all the quantities but $\overset{e}{}_{1,0}$; *change generally* $\overset{\dot{e}}{}_{0,\mu}$ *into* $\overset{\dot{e}}{}_{0,\mu+1}$, *and take the fluent with respect to this last. The same terms must be kept only once.*

By this rule, we find $\underset{m,n}{\mathrm{B}}$ beginning with $\varphi^{\prime\prime\ldots(m+n)}(c) \times \frac{\left(\overset{c}{}_{1,0}\right)^m}{2.3\ldots m} \times \frac{\left(\overset{c}{}_{0,1}\right)^n}{2.3\ldots n}$

for origin of derivation: the reader may compare it with that given by M. Arbogast at p. 113 of his work.

21. If the function to be expanded contains functions of many double multinomials, all the formulas, and rules, that have been given for one, may be extended to this case, by means of equations (κ) and (λ); in the same manner as a like extension was made in treating of simple multinomials.

Thus, from the method of finding $\underset{m,n}{\mathrm{B}}$ given in article 19, we get the following

Rule.

To find $\underset{m,n}{\mathrm{B}}$ *from* $\underset{m+1,n-1}{\mathrm{B}}$ *in the expansion of any function of any functions of the double multinomials*

$$\begin{array}{ccccc} c + \overset{c}{}_{1,0}x + & ; & e + \overset{e}{}_{1,0}x + & ; & d + \overset{d}{}_{1,0}x + \text{ ; \&c.} \\ + \overset{c}{}_{0,1}y + & & + \overset{e}{}_{0,1}y + & & + \overset{d}{}_{0,1}y + \\ + & & + & & + \end{array}$$

1*st. Consider only the c's, and take the fluxion of* $\underset{m+1,n-1}{\mathrm{B}}$ *with respect to all of them except* $\overset{e}{}$, $\overset{c}{}_{0,1}$, $\overset{c}{}_{0,2}$, &c.; *and proceed exactly in the same manner as was directed for one double multinomial in article* 19.

2*dly. Neglect all the terms in* $\underset{m+1,n-1}{\mathrm{B}}$ *which contain any of the c's but c; and proceed, with the remaining terms, in the same manner with respect to the e's.*

3dly. Neglect all terms in $\underset{m+1,\,n-1}{\mathrm{B}}$ *which contain any of the c's or e's except c and e; and proceed, with the remaining terms, in the same manner with respect to the d's. And so on according to the number of multinomials.*

The sum of the terms, thus obtained, will give $\underset{m,n}{\mathrm{B}}$.

It is scarcely necessary to observe that, when we have got a few of the higher terms, by this rule, the preceding ones may be found from the equation

$$\underset{m-\mu,\,n-\nu}{\mathrm{B}} = \left(\frac{\overset{\mu,\nu}{\mathrm{B}}_{m,n}}{\underset{\mu,\nu}{\Delta}}\right)_{c}$$

as in the case of one double multinomial.

To find any coefficient, without a knowledge of the rest, when the function contains more than one double multinomial, we must combine the rule in the last article, with what was shewn in article 14.*

22. Thus we have a complete and simple theory of the expansion of functions of double multinomials; and from equations (ϰ) and (λ) a precisely similar theory may be derived for multinomials of higher kinds.

But it is wholly unnecessary to enter into further details; we are able, without any more trouble, to see what must be the solution of the following

General Problem.

It is required, in the expansion of any function of a multinomial of any kind, to find $\underset{m,\,n,\,r,\,s,\,t,\,\&c.}{\mathrm{B}}$ *the coefficient of* $x^m y^n z^r u^s v^t$ *&c. from* $\underset{m+1,\,n-1,\,r,\,s,\,t,\,\&c.}{\mathrm{B}}$ *that of* $x^{m+1} y^{n-1} z^r u^s v^t$ *&c.*

* See Note II.

Rule.

Take the fluxion of the latter with respect to all the quantities except $^{c}, {}^{c}_{0,\mu,}, {}^{c}_{0,\mu,\nu,}, {}^{c}_{0,0,\rho,}$ *&c that have nothing on the left hand of the first comma; change generally* $\overset{c}{}_{\mu,\nu,\rho,\&c.}$ *into* $\overset{c}{}_{\mu-1,\nu+1,\rho,\&c.}$ *and take the fluent with respect to this last. The same terms must be kept only once.*

The extension of this to any number of multinomials is exactly the same as the similar extension, for double multinomials, in the last article.

Second General Problem.

93. *It is required, in the case of the last problem, to find* $B^{m,n,r,s,t,\&c.}$ *without a knowledge of any other coefficient.*

This will be accomplished if we can find $\Downarrow^{\mu\ldots(r-1)}$ *which multiplies* $\varphi(c)^{\mu\ldots(r-1)}$ *from* $\Downarrow^{\mu\ldots r}$ *which multiplies* $\varphi(c)^{\mu\ldots r}$.

Rule.

1*st. Take the fluxional coefficient of* $\Downarrow^{\mu\ldots r}$ *with respect to* $\overset{c}{}_{1,0,0,0,0,\&c.}$ *and of this fluxional coefficient take the fluxion with respect to all the quantities; change generally* $\overset{c}{}_{\mu,\nu,\rho,\&c.}$ *into* $\overset{c}{}_{\mu+1,\nu,\rho,\&c.}$ *and take the fluent with respect to this last.*

2*dly. If there be any terms in* $\Downarrow^{\mu\ldots r}$ *in which the unit under* $\overset{c}{}_{1,0,0,0,0,\&c.}$ *if it be one of the factors, is the only left hand figure, they will require a further process.*

Take the fluxional coefficient with respect to $\overset{c}{}_{0,1,0,0,0,\&c.}$ *and of this take the fluxion with respect to all the quantities, except*

$\overset{\epsilon}{}_{1,0,0,0,0,\&c.}$; *change generally* $\overset{\epsilon}{}_{0,\mu,\nu,\rho,\&c.}$ *into* $\overset{\epsilon}{}_{0,\mu+1,\nu,\rho,\&c.}$ *and take the fluent.*

3*dly. Any terms, in* $\overset{n\ldots r}{\psi}$, *in which the units under* $\overset{\epsilon}{}_{1,0,0,0,0,\&c.}$ *and* $\overset{\epsilon}{}_{0,1,0,0,0,\&c.}$, *if they are amongst the factors, are the only figures in the first and second left hand places, will require a still further process. Take the fluxional coefficient with respect to* $\overset{\epsilon}{}_{0,0,1,0,0,\&c.}$, *and of this take the fluxion with respect to all the quantities except* $\overset{\epsilon}{}_{1,0,0,0,0,\&c.}$ *and* $\overset{\epsilon}{}_{0,1,0,0,0,\&c.}$; *change generally* $\overset{\epsilon}{}_{0,0,\mu,\nu,\rho,\&c.}$ *into* $\overset{\epsilon}{}_{0,0,\mu+1,\nu,\rho,\&c.}$ *and take the fluent.*

The rule will proceed in this manner, till it contains n *parts if the multinomial be of the* nth *order. The terms arising from all these parts must be added, and the same terms kept only once.*

24. In treating multinomials of higher kinds, I have given rules by which certain terms are frequently found more than once: this was done for the sake of simplicity, and that the precepts might be easily retained in the memory; but was by no means a matter of necessity; for rules might without difficulty have been formed (as from equations (ν) and (ξ) for a double multinomial) by which no superfluous terms would have been found.

25. It will not be an improper termination of this paper, to state what are the peculiar advantages of the method pursued in it.

To many, I have no doubt, its brevity will be a recommendation; and that it requires no notation different from that in common use.

For though I have represented some of the fluxional coeffi-

cients in an unusual manner, as $\left(\frac{\dot{B}}{\frac{n-1}{\dot{c}\dot{c}}}\right)$ by $\left(\frac{\dot{B}}{\frac{n-1}{\dot{c}^2}}\right)^c$ the doing so was not necessary; but it appeared advisable to make a distinction between the taking the fluxion with respect to c, and the same operation with respect to $\dot{c}$, $\ddot{c}$, &c. which enter into the coefficients in a manner different from the first.

The uniformity of the procedure is such, that, when we have arrived at the rules for one simple multinomial, a person of any skill in this kind of inquiry might easily divine those for the more difficult cases. But the most important circumstance is the perfection given to *inverse derivation*, and the facility with which we may, by that means, find any large number of terms in the expansion of the higher kinds of multinomials, as has been shewn in article 18 and 19.

The last advantage I shall notice is, that the same rules of derivation serve equally for the expansion of a function of one or of a thousand multinomials: whereas, from M. ARBOGAST's methods, it would not, I imagine, be very easy to give a rule in words for the expansion of a function of five or six.

NOTES.

Note I. The rule in article 8 may be differently enunciated thus.

To find $\overset{n\ldots(m-1)}{\downarrow}$ *take the fluxional coefficient of* $\overset{n\ldots m}{\downarrow}$ *with respect to* c; *and of this fluxional coefficient take the fluxion with respect to the last quantities; change generally* $c^{n\ldots m}$ *into* $c^{n\ldots(m+1)}$ *and take the fluent with respect to this last.*

2dly. If the last quantity but one be that which precedes the last in the number of strokes make it vary in the same manner and take the fluent.

This is simpler than the rule in article 8, and more conformable to the mode of expression made use of in other parts of the paper.

Note II. In looking back on what I have written, I am apprehensive it may be thought that I have affected too great brevity in the last paragraph of article 21. That the reader may have no difficulty, the following problem is added, to illustrate what was said in the passage alluded to.

Problem.

To find at once $B^{m,n}$ *in the expansion of a function of two functions of double multinomials.*

It is plain that $B^{m,n}$ must contain all the possible combinations of c's and e's (see the notation of article 21) that can be formed with this condition; that the number of left hand strokes be m; the number of right hand strokes n. Every rth power must be divided by the product $2.3.4\ldots r$. And the fluxional coefficient $\varphi^{\alpha,\beta}$, that multiplies each term, will have, for the left hand figure and over it, the sum of the exponents of the c's in that term; for the right hand figure β the sum of the exponents of the e's.

Now to get all the combinations of the kind mentioned above, with their proper divisors, we must plainly take, for origins of derivation, all the terms of the following product, when actually multiplied.

$$\left\{\frac{\left(c_{1,0}\right)^m}{2.3\ldots m}+\frac{\left(c_{1,0}\right)^{m-1}}{2.3\ldots(m-1)}\times e_{1,0}+\frac{\left(c_{1,0}\right)^{m-2}}{2.3\ldots(m-2)}\times\frac{\left(e_{1,0}\right)^2}{2}+\ldots\ldots+\frac{\left(e_{1,0}\right)^m}{2.3\ldots m}\right\}$$

multiplied by

$$\left\{\frac{\left(c_{0,1}\right)^n}{2.3\ldots n}+\frac{\left(c_{0,1}\right)^{n-1}}{2.3\ldots(n-1)}\times e_{0,1}+\frac{\left(c_{0,1}\right)^{n-2}}{2.3\ldots(n-2)}\times\frac{\left(e_{0,1}\right)^2}{2}+\ldots\ldots+\frac{\left(e_{0,1}\right)^n}{2.3\ldots n}\right\}$$

Suppose any one of these origins to be

$$\frac{\binom{c}{1,0}^r}{2.3\ldots r} \times \frac{\binom{c}{0,1}^s}{2.3\ldots s} \times \frac{\binom{e}{1,0}^t}{2.3\ldots t} \times \frac{\binom{e}{0,1}^u}{2.3\ldots u} \ldots\ldots\ldots (A).$$

Let Δ, Δ^2, Δ^3, &c. represent the successive derivations made according to the rule in article 20. It is plain that all the terms got from the origin of derivation (A) will be expressed by the product

$$\left\{\frac{\binom{c}{1,0}^r}{2.3\ldots r} \times \frac{\binom{c}{0,1}^s}{2.3\ldots s} + \Delta\left\{\frac{\binom{c}{1,0}^r}{2.3\ldots r} \times \frac{\binom{c}{0,1}^s}{2.3\ldots s}\right\} + \Delta^2\left\{\frac{\binom{c}{1,0}^r}{2.3\ldots r} \times \frac{\binom{c}{0,1}^s}{2.3\ldots s}\right\} + \&c.\right\}$$

multiplied by

$$\left\{\frac{\binom{e}{1,0}^t}{2.3\ldots t} \times \frac{\binom{e}{0,1}^u}{2.3\ldots u} + \Delta\left\{\frac{\binom{e}{1,0}^t}{2.3\ldots t} \times \frac{\binom{e}{0,1}^u}{2.3\ldots u}\right\} + \Delta^2\left\{\frac{\binom{e}{1,0}^t}{2.3\ldots t} \times \frac{\binom{e}{0,1}^u}{2.3\ldots u}\right\} + \&c.\right\}.$$

In this manner may the terms be derived from all the origins; after which we have only to arrange them under their appropriate fluxional coefficients.

If we wanted to find immediately B in a function of two multinomials of a *m*, *n*, *r*, &c.

still higher kind, the method would be exactly similar.

Note III. In the preceding pages, I have considered the expansion of multinomial functions *generally*; and abstained from giving particular examples, that the paper might not be extended to an unreasonable length. There are, however, some cases, —*when the function is a whole positive power*—which require a separate notice. The method of direct derivation given in article 5, and a similar one at the end of article 11 will here fail: this indeed is of no consequence, as the rules in article 6 and 12 are both easier than the former, and applicable to every case. But it will be necessary to give new methods of inverse derivation; for if we consider those in the paper, in article 7 for example, it will easily appear, that though they are true generally for the *m*th power, the case is very different when we give to this letter the particular values 1, 2, 3, &c. The reason of which is that the fluxional coefficients of $f(c)$, after the first, or the second, or the third, &c. vanish; and these functions may be said not improperly, when compared with the general form, to give *defective expansions*; any rules, therefore, which depend on the *depression* of the fluxional coefficients of $f(c)$ will be of no use here.

The following very extensive rule is the reverse of that, for direct derivation, in article 12. It agrees, in its *simplest case*, with that of M. Arbogast in his article 36.

Rule.

To find $\underset{n-1}{B}$ from B in a function of any functions of the multinomials $c + \overset{\prime}{c}x + \overset{\prime\prime}{c}x^{n} +$, $e + \overset{\prime}{e}x + \overset{\prime\prime}{e}x^{n} +$, $d + \overset{\prime}{d}x + \overset{\prime\prime}{d}x^{n} +$, &c. 1st. Consider only those terms, in B, which contain some of the quantities $\overset{\prime}{c}, \overset{\prime\prime}{c}, \overset{\prime\prime\prime}{c}$, &c.; reject all the terms in which the last of these letters are raised to a higher than the first power: reject also *(if there be more than one multinomial)* such terms as contain none of the above mentioned quantities but the first power of $\overset{\prime}{c}$. Change, generally, in each remaining term, the last of the c's as $\overset{\prime\prime\prime\ldots m}{c}$ into $\overset{\prime\prime\ldots(m-1)}{c}$, and take the fluent with respect to this quantity.

2dly. Neglecting those terms, in $\underset{n}{B}$, into which $\overset{\prime}{e}, \overset{\prime\prime}{e}, \overset{\prime\prime\prime}{e}$, &c. enter, consider those, of the remainder, which contain $\overset{\prime}{e}, \overset{\prime\prime}{e}, \overset{\prime\prime\prime}{e}$, &c. rejecting all those terms in which the last of the e's are raised to a higher than the first power. Those terms must also be rejected *(if there be more than two multinomials)* which contain none of the e's but the first power of $\overset{\prime}{e}$. Change generally, in the remaining terms, the last of the e's as $\overset{\prime\prime\ldots m}{e}$ into $\overset{\prime\prime\ldots(m-1)}{e}$, and take the fluent, with respect to this last.

3dly. Neglecting the terms into which $\overset{\prime}{c}, \overset{\prime\prime}{c}, \overset{\prime\prime\prime}{c}$, &c. $\overset{\prime}{e}, \overset{\prime\prime}{e}, \overset{\prime\prime\prime}{e}$, &c. enter, consider those, of the remainder, which contain $\overset{\prime}{d}, \overset{\prime\prime}{d}, \overset{\prime\prime\prime}{d}$, &c. and proceed as before.—And so on.

This rule has no difficulty, whatever may be the number of multinomials.

The words in italics, were inserted to make the rule include the finding of B from $\underset{1}{B}$; they are of no use when n is greater than one.

Similar rules for multinomials of higher orders are formed with equal ease; being the reverse of those that have been given for direct derivation.

IV. *On a Case of nervous Affection cured by Pressure of the Carotids; with some physiological Remarks.* By C. H. Parry, *M. D. F. R. S.*

Read 20th December, 1810.

Observing that the Royal Society, of which I have the honour to be a Member, occasionally receives communications illustrative of the laws of animal life, which are indeed the most important branch of physics, I take the liberty of calling their attention to a case, confirming a principle which I long ago published, and which, I believe, had never till then been remarked by pathologists.

About the year 1786, I began to attend a young lady, who laboured under repeated and violent attacks, either of head-ach, vertigo, mania, dyspnoea, convulsions, or other symptoms usually denominated nervous. This case I described at large to the Medical Society of London, who published it in their Memoirs, in the year 1788. Long meditation on the circumstances of the case, led me to conclude, that all the symptoms arose from a violent impulse of blood into the vessels of the brain; whence I inferred, that as the chief canals conveying this blood were the carotid arteries, it might perhaps be possible to intercept a considerable part of it so impelled, and thus remove those symptoms which were the supposed effect of that inordinate influx. With this view, I compressed with my thumb one or both carotids, and uniformly found all the symptoms removed by that process. Those circumstances of

rapidity or intensity of thought, which constituted delirium, immediately ceased, and gave place to other trains of a healthy kind; head-ach and vertigo were removed, and a stop was put to convulsions, which the united strength of three or four attendants had before been insufficient to counteract.

That this extraordinary effect was not that of mere pressure, operating as a sort of counteracting stimulus, was evident: for the salutary effect was exactly proportioned to the actual pressure of the carotid itself, and did not take place at all, if, in consequence of a wrong direction either to the right or left, the carotid escaped the effects of the operation.

This view of the order of phœnomena was, in reality, very conformable to the known laws of the animal œconomy. It is admitted, that a certain momentum of the circulating blood in the brain, is necessary to the due performance of the functions of that organ. Reduce the momentum, and you not only impair those functions, but, if the reduction go to a certain degree, you bring on syncope, in which they are for a time suspended. On the other hand, in nervous affections, the sensibility and other functions of the brain are unduly increased; and what can be more natural than to attribute this effect to the contrary cause, or excessive momentum in the vessels of the brain? If, however, this analogical reasoning has any force in ascertaining the principle, I must acknowledge that it did not occur to me till twenty years afterwards, when a great number of direct experiments had appeared to me clearly to demonstrate the fact.

From various cases of this kind, I beg leave to select one which occurred to me in the month of January, 1805.

Mrs. T. aged 51, two years and a half beyond a certain

critical period of female life, a widow, mother of two children, thin, and of a middle size, had been habitually free from gout, rheumatism, haemorrhoids, eruptions, and all other disorders, except those usually called nervous, and occasional colds, one of which, about two years and a half before, had been accompanied with considerable cough, and had still left some shortness of breathing, affecting her only when she used strong muscular exertion, as in walking up stairs, or up hill.

In February 1803, after sitting for a considerable time in a room without a fire, in very severe weather, she was so much chilled as to feel, according to her own expression, "as if her "blood within was cold." In order to warm herself, she walked briskly for a considerable time about the house, but ineffectually. The coldness continued for several hours, during which she was seized with a numbness or sleepiness of her left side, together with a momentary deafness, but no privation or hebetude of the other senses, or pain or giddiness of the head. After the deafness had subsided, she became preternaturally sensible to sound in the ear of the affected side, and felt a sort of rushing or tingling in the fingers of the left hand, which led her to conclude that "the blood went "too forcibly there."

Though the coldness went off, what she called numbness still continued, but without the least diminution of the power of motion in the side affected. In about six weeks, the numbness extended itself to the right side.

Among various ineffectual remedies for these complaints, blisters were applied to the back, and the inside of the left arm above the elbow. The former drew well. The latter inflamed without discharging: so that a poultice of bread and

milk was put on the blistered part. After this period, the muscles of the humerus began to feel as if contracted and stiff; and these sensations gradually spread themselves to the neck and head, and all across the body, so as to make it uncomfortable for her to lie on either side, though there was no inability of motion.

She now began to be affected with violent occasional flushings of her face and head, which occurred even while her feet and legs were cold, together with a rushing noise in the back of the head, especially in hot weather, or from any of those causes which usually produce the feelings of heat.

It is difficult to give intelligible names to sensations of a new and uncommon kind. That, which this lady denominated numbness, diminished neither the motion nor the sensibility of the parts affected. It was more a perception of tightness and constriction, in which the susceptibility of feeling in the parts was in fact increased; and the skin of the extremities was so tender, that the cold air produced a sense of uneasiness, the finest flannel or worsted felt disagreeably coarse, and the attempt to stick a pin with her fingers caused intolerable pain.

In the month of September 1803, not long after the application of the blisters, she experienced in certain parts of the left arm and thigh, that sensation of twitching which is vulgarly called the "life blood," and which soon extended itself to the right side. Shortly afterwards, she began to perceive an actual vibration or starting up of certain portions of the flexor muscles of the fore-arm, and of the deltoid on the left side; not so, however, as to move the arm or hand.

This disorder had continued with little variation to the

period of my first visit. The vibrations constantly existed while the arm was in the common posture, the fore-arm and hand leaning on the lap. If the arm were stretched strongly downwards, the vibration of the flexors ceased, but those of the deltoid continued. The arm being strongly extended forwards, all ceased; but returned as soon as the muscles were relaxed. The vibrations were of different degrees of frequency, and at pretty regular intervals, usually about 80 in a minute. They were increased in frequency and force by any thing which agitated or heated the patient, and were always worse after dinner than after breakfast. The pulse in the radial artery was 80 in a minute, and rather hard. That in the carotids was very full and strong; and each carotid appeared to be unusually dilated for about half an inch in length, the adjacent portions above and below being much smaller, and of the natural size. I much regret that I find in my notes of this case, no inquiry whether there was any coincidence between the systoles of the heart, and the muscular vibrations. The patient's feet were usually cold, and her head and face hot. The feeling in her limbs was much as I have above described, except that the sensibility was somewhat less acute than it had been, and she complained of a tightness all over her head, as if it had been bound with a close night-cap. Her sleep was usually sound on first going to bed, but afterwards, for the most part, interrupted by dreaming. Bowels generally costive: appetite moderate: no flatulency or indigestion: tongue slightly furred, without thirst: urine variable, but generally pale.

The late Mr. GEORGE CROOK, surgeon, was present while I made these examinations; and when we afterwards con-

versed together, I remarked to him, that if my theory of the usual cause of spasmodic or nervous affections were well founded, I should probably be able to suppress or restrain these muscular vibrations of the left arm, by compressing the carotid artery on the opposite or right side; while little effect might perhaps be produced, by compressing the carotid of the side affected. The event was exactly conformable to my expectation. Strong pressure on the right carotid uniformly stopped all the vibrations, while that on the left had no apparent influence. I may add that these experiments were afterwards, at my request, repeated on this lady in London by Dr. BAILLIE, and, as he informed me in a letter, with a similar result.

It is perfectly well known to many of the learned Members of this Society, that irritations of the brain, when of moderate force, usually exhibit their effects on the nerves or muscles of the opposite side of the body; and in the case before us, it is difficult to understand how the suspension of these automatic motions could have been produced by this pressure of the opposite carotid, in any other way than by the interruption of the excessive flow of blood through a vessel morbidly dilated; in consequence of which interruption, the undue irritation of the brain was removed, and the muscular fibres permitted to resume their usual state of rest.

From these and many other similar facts, I am disposed to conclude, that irritation of the brain, from undue impulse of blood, is the common though not the only cause of spasmodic and nervous affections; and I can with the most precise regard to truth add, that a mode of practice, conformable to this principle has enabled me, during more than twenty years, to cure

a vast number of such maladies, which had resisted the usual means.

An investigation of all the modifications of the principle itself, and of its numerous relations to Therapeutics, would be inconsistent with the views of the ROYAL SOCIETY, and must be reserved for another place.

Bath, Dec. 8, 1810.

V. *On the Non-existence of Sugar in the Blood of Persons labouring under Diabetes Mellitus. In a Letter to* Alexander Marcet, *M. D. F. R. S. from* William Hyde Wollaston, *M. D. Sec. R. S.*

Read January 24, 1811.

My dear Sir,

In reply to your inquiry respecting my experiments upon the non-existence of sugar in the serum of diabetic persons, which I have mentioned to you at different periods, I am really ashamed to reflect how long I have suffered them to remain neglected, when I consider their tendency to elucidate a curious point of physiological research.

My first endeavours to detect sugar in the serum of the blood were made soon after perusing the second edition of Dr. Rollo's Treatise on the Diabetes (which was published in 1798,) at the request of Dr. Baillie, who was so obliging as to furnish me with various specimens of diabetic blood and serum for this purpose.

The other set of experiments which I made with reference to the same question were not thought of till the following year. The inquiry was then left unfinished, and I never resumed it; for as I soon after* relinquished the practice of physic, I desisted in a great measure from prosecuting any inquiries connected with medicine.

However, since so much of this subject as is strictly physio-

* In 1800.

logical, relating to the natural course of circulating fluids, and more especially so much of the investigation as is conducted by chemical means, is within the range of those pursuits which are generally interesting to the Royal Society, I will endeavour to give you as distinct an account as I am able of the progress of my own experiments, requesting that you will in return state, more fully than you have hitherto done, the result of that further step in the inquiry which you took at my suggestion, and if it is agreeable to you, we will without delay make a joint communication of our researches to the Society.

Although Dr. ROLLO had been assisted in the chemical part of his inquiry by the well known talents of Mr. CRUICKSHANK, it appears that they "had not been so fortunate as to obtain "a sufficient quantity of serum for chemical experiment;"* and were unable fully to satisfy themselves by the taste or by other means which they could employ, concerning the existence or non-existence of sugar in the blood of persons labouring under diabetes; but nevertheless they were persuaded of its presence.

For the purpose of forming some judgment on this question, Mr. CRUICKSHANK made trial of the quantities of oxalic acid that could be formed from serum or from blood in their natural state, and from the same serum or blood after the addition of a certain proportion of sugar; and from the difference perceptible in these trials, he formed a probable conjecture respecting the presence or absence of sugar in the serum of diabetic persons.

This method, it is evident, is liable to a two-fold objection: first, that an excess of other ingredients beside sugar will

* ROLLO on Diabetes, p. 408.

cause an increase of the quantity of oxalic acid formed, and secondly, that slight variations in the process for forming oxalic acid will unavoidably occasion differences in the result.

The method which I employed appears to me capable of detecting much smaller quantities of such an ingredient, for though it might not enable us to distinguish exactly the nature of any small quantity that may be discovered, still the mere question of absence or presence admits of determination with great precision.

For this purpose I investigated, in the first place, how the albuminous part of healthy serum could be most completely coagulated, and by what appearances the presence of sugar that had been added to it would be most easily discerned.

When heat alone had been employed for the coagulation of serum, to which water had been added, that which exsuded from it was still found to contain a portion of albumen dissolved in it, and if this were allowed to remain, any saccharine matter which might be present would be disguised, and could not with certainty be detected.

I found, however, that this residuum of coagulable matter might be altogether prevented by the addition of a small quantity of dilute acid to the serum before coagulation.* To six drams of serum I added half a dram of muriatic acid previously diluted with one dram and a half of water, and immersed the phial containing them in boiling water during four minutes. The coagulation was thus rendered complete. In the course of a few hours a dram or more of water exsudes from serum that has been so coagulated. If a drop of this water be eva-

* I presumed that this portion of albumen was retained in solution by the alkali redundant in serum, and added the acid for the purpose of neutralizing it.

porated, the salts which it contains are found to crystallize, so that the form of the crystals may be easily distinguished; they are principally common salt.

If any portion of saccharine matter has been added to the serum previous to coagulation, the crystallization of the salts is impeded, or wholly prevented, according to the quantity of sugar present.

If the quantity added does not exceed two grains and a half to the ounce, the crystallization is not prevented; but even this small quantity is perceptible by a degree of blackness that appears after evaporation: occasioned, as I suppose, by the action of a small excess of acid on the sugar.

If five grains have been added, the crystallization is very imperfect, and soon disappears in a moist air by deliquescence of the sugar. The blackness is also deeper than in the former case.

By addition of ten grains to the ounce, the crystallization of the salts is entirely prevented, and the degree of blackness, and disposition to deliquesce are of course more manifest than with smaller quantities.*

As I was aware that the sugar obtained from diabetic urine is a different substance from common sugar (approaching more nearly to the sugar of figs), I had the precaution to repeat the same series of experiments upon serum, to which I made corresponding additions of dry sugar, that I had formerly extracted from the urine of a person who voided it in considerable quantity; and I found the effects to be perfectly similar in every respect.

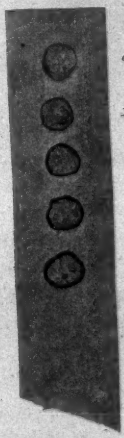
* In the annexed Plate are represented the degrees of blackness of the drop occasioned by adding one, two, three, and four grains of sugar to six drachms of serum.

As a further test of the absence or presence of sugar, I found it convenient to add a little nitric acid to the salts that remained after crystallization of the drop. If the serum has been successfully coagulated without any addition of sugar, the addition of nitric acid merely converts the muriatic salts into nitrates, and nitrate of soda is seen to crystallize without foam or blackness. But when sugar has been added, a white foam rises round the margin of the drop, and if further heat be applied, it becomes black in proportion to the quantity of sugar present.

Such are the appearances when the proportions have been duly adjusted, and the proper heat for coagulation applied. I must own, however, that I could not always succeed to my satisfaction at the time when these experiments were conducted, and I am inclined to ascribe occasional failures to having used more muriatic acid than was really necessary, which by excess of heat might redissolve a part of the coagulated albumen, and thence occasion appearances, which, without careful discrimination, might be ascribed to sugar.

After having, by this course of experiment, satisfied myself as to the phenomena exhibited by serum in its natural state, and the effects of any small additions of sugar, I then proceeded to the examination of such specimens of diabetic blood or of serum, as I was able to procure.

The first which I examined was a portion of blood that had been taken from a person whose urine had been analysed, and found to contain sugar. This blood had been dried, when fresh, by a gentle heat, so as not to coagulate the serum. After being reduced to powder, it was mixed with water, in order that every thing which remained soluble might be ex-


0
1
2
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tracted. A little muriatic acid was then added, and sufficient heat applied for coagulation of the albumen. The water that separated after coagulation was found to contain the salts of the blood, but no trace whatever of sugar.

A second specimen of dried blood, that had been ascertained to be diabetic on the same evidence as the preceding, was examined in a similar manner, with the same result, as no appearance of sugar could be discerned.

In a third instance, I had some serum from the blood of a person whose urine had been tasted, and found "*very sweet.*" (I had no opportunity of procuring any of this urine for analysis). After a portion of this serum had been coagulated, with the addition of the usual proportion of muriatic acid, there was no appearance whatever of sugar. But when three grains of diabetic sugar had been added to another ounce of the same serum, the presence of this quantity was manifest by the same process.

I had also a fourth opportunity of examining serum of a person whose urine contained so much saccharine matter, that an ounce of it yielded, by evaporation, thirty-six grains of extract. In this instance I was not so successful in my experiment; for, though I was satisfied that no sugar was present, there certainly was a degree of blackness, which might have been occasioned by about one grain and a half of sugar in the ounce of serum. But this black matter appeared not to be sugar: it was more easily dried than sugar: it was not fusible by heat as sugar is: and its refractive power* was too great for that of sugar.

* The method by which this was tried has since that time been described in the Philosophical Transactions for 1802.

I unfortunately had no opportunity of repeating the experiment on a second portion of the same serum, having inconsiderately employed it for other experiments, and coagulated it at the same time with the former.

In the next experiment I added half a dram of the urine of the same person to six drams of the serum and with a due proportion of diluted muriatic acid coagulated as before. Although the quantity of extract added did not exceed $\frac{1}{16}$, or two grains and a quarter of extract, the difference was very manifest by the darkness of the colour and the defective crystallization of the salts.

To the remaining quantity of the serum I had added twice the former proportion of the urine, and found that this quantity did not wholly prevent the crystallization of the salts during the evaporation of the drop.

The result of these trials was such, as to satisfy me that the serum in this instance contained no perceptible quantity of sugar, or at least that the water separable from the coagulated serum did not contain one-thirtieth part of that proportion which I had found in the urine of the same person.

In order to account for the presence of sugar in the urine, we must consequently either suppose a power in the kidneys of forming this new product by secretion, which does not seem to accord with the proper office of that organ; or, if we suppose the sugar to be formed in the stomach by a process of imperfect assimilation, we must then admit the existence of some channel of conveyance from the stomach to the bladder, without passing through the general system of blood-vessels. That some such channel does exist, Dr. DARWIN* endeavoured

* Account of the retrograde Motion of the absorbent Vessels, by CHARLES DARWIN.

to ascertain, by giving large doses of nitre, which he could perceive to pass with the urine, but could not detect in its passage through the blood; and he imagined the channel by which it was conveyed to be the absorbent system, upon the supposition that they might admit of a retrograde motion of their contents.

Without adopting the theory of Dr. DARWIN, it did appear to me that the fact deserved to be ascertained by some test more decisive than nitre, and I conceived that if prussiate of potash could be taken with safety, its presence would be discerned by means of a solution of iron in as small proportion as almost any known chemical test. Upon trial of this salt, I found that a solution of it might be taken without the least inconvenience, and that in less than one hour and a half the urine became perceptibly impregnated, and continued so to the fifth or sixth hour, although the quantity taken had not amounted to more than three grains of the salt.

After a few previous trials of the period when the principal impregnation of the urine might be expected, and when the presence of the prussiate (if it existed in the blood) might with most reason be presumed to occur, a healthy person about thirty-four years of age was induced to take a dose corresponding to three grains and a half of the dry salt, and to repeat it every hour to the third time. The urine being examined every half hour, was found in two hours to be tinged, and to afford a deep blue at the end of four hours. Blood was then taken from the arm, and the coagulum, after it had formed, was allowed to contract, so that the serum might be fully separated. The presence of the prussiate was then endeavoured to be discovered by means of a solution of iron, but

without effect; and as I thought that the redundant alkali (which had been ascertained to prevail in this serum) might tend to prevent the appearance of the precipitate, I added a small quantity of dilute acid; but still I could not discern that any degree of blueness was occasioned by it.

This experiment having been repeated a second time with the same result, seemed to me nearly conclusive with respect to the existence of some passage, by which substances certainly known to be in the stomach may find their way to the bladder without being mixed with the general mass of circulating fluids.

Being desirous of ascertaining whether the prussiate could be discovered in any other secretions, I have repeatedly examined my saliva, at times when the urine has manifested a very strong blue, by adding solution of iron, but I could at no time perceive the saliva to be tinged.

I have also, during a severe cold, accompanied with profuse running of water from the nose, made a similar examination of this discharge, but have not been able to perceive any trace of the prussic acid.

It was nearly in this state that I left the inquiry at the period I have mentioned, and I do not remember to have made any other experiments, when I requested your assistance in making trial of the serum that is secreted in consequence of the application of a blister. Your report upon the result of your experiments, in addition to those which I have above related, nearly satisfied me as to the existence of some unknown channel of conveyance by which substances may reach the bladder.

With respect to Dr. DARWIN's conception of a retrograde

action of the absorbents, it is so strongly opposed by the known structure of that system of vessels, that I believe few persons will admit it to be in any degree probable.

Since we have become acquainted with the surprising chemical effects of the lowest states of electricity, I have been inclined to hope that we might from that source derive some explanation of such phenomena. But though I have referred* secretion in general to the agency of the electric power with which the nerves appear to be indued, and am thereby reconciled to the secretion of acid urine, from blood that is known to be alkaline, which before that time seemed highly paradoxical, and although the transfer of the prussiate of potash, of sugar, or of other substances may equally be effected by the same power as acting cause, still the channel through which they are conveyed remains to be discovered by direct experiment.

I have, indeed, conjectured that, by examining the blood in the abdominal vessels, or contents of the lacteals, it might be possible to detect them *in transitu*; but I have not been inclined to make such experiments on living animals, as would perhaps throw light upon the subject.

I remain, Dear Sir,
with great regard,
yours very truly,

W. H. WOLLASTON.

January 1, 1811.

* Philosophical Magazine for June 1809.

Reply of Dr. MARCET *on the same Subject.*

Russell Square, January 9, 1811.

MY DEAR SIR,

I AM much gratified to find that you have at last been induced to communicate to the Royal Society your curious inquiry respecting the state of the blood in diabetes. I was anxious that the specious hypothesis of the presence of sugar in diabetic blood, which had been sanctioned by the authority of Dr. ROLLO and Mr. CRUICKSHANK, and which I had myself urged in support of their theory, fourteen years ago, in an inaugural publication, should no longer obtain an undue weight amongst physiological inquirers.

With regard to the experiments which I tried at your request some years ago, with a view to ascertain whether prussiat of potash taken into the stomach, and found to exist in the urine, could also be detected in other secretions, I find, on referring to my memorandums, the following particulars which I shall transcribe verbatim.

" August 19, 1807. Having heard from Dr. WOLLASTON, that prussiat of potash could be taken into the stomach with perfect safety, and that its presence could afterwards be discovered in the urine, but not in the serum; and being invited by him to follow up this inquiry, with a view to connect it with the theory of diabetes, I tried the following experiments.

Experiment 1.

"After having satisfied myself, by trials made by some medical gentlemen upon themselves, that considerable doses of prussiat of potash might be taken without the least inconvenience, I gave to a young woman labouring under diabetes mellitus, five grains of prussiat of potash dissolved in water, and this was repeated every hour till she had taken thirteen or fourteen such doses. After the fifth dose, her urine, by the addition of a drop or two of a solution of sulphat of iron, turned blue instantly. At this period of the experiment, a blister was applied to her stomach, and after a few hours, whilst still taking the prussiat of potash, and whilst the urine strongly indicated its presence, the blister was cut and the serum collected. This serous fluid being, in the same manner as the urine, subjected to the action of a solution of sulphat of iron, did not suffer any change of colour in the least indicative of the presence of prussic acid. Yet the urine still remained capable of imparting a blue colour to solution of iron, fifteen hours after taking the last dose of the prussiat of potash.

Experiment 2.

"The same person being soon afterwards put upon a course of ferruginous medicines, and having taken considerable quantities of sulphat of iron, an idea naturally occurred to me that the phenomenon might perhaps be reversed; but upon adding prussiat of potash to the urine, no vestige of iron could be discovered, and the same attempt was repeated several times with the same negative result.

Experiment 3.

"Dec. 2, 1807. The fluid obtained by means of a blister (as in Experiment 1,) being not immediately derived from the circulation, since it may be considered as the product of a secretion, I was desirous of repeating Dr. WOLLASTON's experiment on the serum itself, under circumstances of impregnation similar to those in which the serum of the blister was examined.

"For this purpose, a young woman after taking, in divided doses, about a dram of prussiat of potash in the course of twelve hours, lost some blood by cupping, an operation which had been ordered for a local complaint under which she laboured. The serum having been allowed to separate, and a little nitric acid having been added to it, not the least vestige of prussic acid appeared in applying the test of sulphat of iron, although the urine made during the six hours which preceded and followed the cupping, was strongly impregnated with that acid, and struck a vivid blue upon adding the smallest quantity of iron."

I have only to observe, in addition to these particulars, that the susceptibility by which prussiat of potash is transmitted to the bladder, seems to vary in different individuals; for in five trials, made at Guy's Hospital in Nov. 1805, I failed of discovering any vestige of that salt in the urine of persons who had taken it in quantities sufficient to produce its appearance in others. Three of these individuals, I should observe, were at the time under mercurial treatment, and an idea occurred to me that

mercury having a great affinity for prussic acid, the presence of that metal in the system might prevent the effect in question. But as in the two other failures, no mercury was present, I cannot lay any stress upon that conjecture. It may be proper to mention, that in the frequent trials which I have made with the prussiat of potash, no symptom or inconvenience whatever has ever occurred which could be ascribed to that salt.

I remain ever,
my dear Sir,
with great esteem, your's sincerely,

ALEX. MARCET.

P.S. Whilst revising the proof of this sheet, it has been observed to me by some friends, and in particular by Dr. HENRY of Manchester, and Dr. R. PEARSON of London, that in order to show distinctly that certain substances find their way to the bladder, without passing through the general circulation, it would be necessary to examine the arterial, as well as the venous blood, since it is not impossible that the whole of the sugar in diabetes, or the prussiat of potash in the experiments above related, may be conveyed to the urinary organs by the arteries, without entering the venous system. According to this hypothesis, it may be conceived that the same substances when conveyed by the arteries to distant parts of the body, may return by the absorbent system, and might in that case be discovered in the thoracic duct. This view of the subject may deserve further investigation; and I hope that this curious question will soon be decided by appropriate experiments.

VI. *On the Rectification of the Hyperbola by Means of Two Ellipses; proving that Method to be circuitous, and such as requires much more Calculation than is requisite by an appropriate Theorem: in which Process a new Theorem for the Rectification of that Curve is discovered.*

To which are added some further Observations on the Rectification of the Hyperbola: among which the great Advantage of descending Series over ascending Series, in many Cases, is clearly shown; and several Methods are given for computing the constant Quantity by which those Series differ from each other. By the Rev. John Hellins, *B. D. F. R. S. and Vicar of Potter's-Pury, in Northamptonshire.*

Being an Appendix to his former Paper on the Rectification of the Hyperbola, inserted in the Philosophical Transactions for the Year 1809. *Communicated by* Nevil Maskelyne, *D.D. F.R.S. Astronomer Royal.*

Read January 10, 1811.

1. The rectification of the hyperbola by means of two ellipses is one proof, among many others, of the great sagacity of the late Mr. John Landen, F.R.S.; and the ingenuity which he displayed on that occasion has obtained the notice and called forth the praises of eminent mathematicians both in this island and on the Continent. Yet, while the great ingenuity of the device is thus generally and justly allowed, this method of rectifying the hyperbola has always appeared to me to be

more curious than useful, as *it is circuitous, and requires much more calculation than is requisite for that purpose by an appropriate theorem.* The establishment of this truth is the main design of this short paper; a truth which, to my surprise, has not been noticed in any book that has come to my hands.*

2. But, before I proceed to investigation, it seems proper to remark, that Mr. LANDEN has, in his *Memoir*† on the Hyperbola and Ellipsis, expressed himself as if he thought that the difference between an hyperbolic arch and its tangent, when both are of an immense length, could not be computed before he published that work. His words at the beginning of the memoir above-mentioned, are these: "Some of the theorems "given by mathematicians for the calculation of fluents by "means of elliptic and hyperbolic arcs requiring, in the appli- "cation thereof, the difference to be taken between an arc of "an hyperbola and its tangent; and such difference being "not directly attainable when such arc and its tangent both "become infinite, as they will do when the *whole* fluent is "wanted, although such fluent be at the same time finite; "those theorems therefore in that case fail, a computation "thereby being then impracticable without some farther "help."

"The supplying that defect I considered as a point of some "importance in geometry, and therefore I earnestly wished "and endeavoured to accomplish that business; my aim being "to ascertain, by means of such arcs as above-mentioned, the "*limit* of the difference between the hyperbolic arc and its

* On this occasion, no one has betrayed more ignorance, nor shown a greater want of candour, than the writer of Art. XIII. in the Monthly Review for April 1803.

† This is the second in the 1st Volume of his *Memoirs*, printed in the year 1780.

" tangent, whilst the point of contact is supposed to be car-
" ried to an infinite distance from the vertex of the curve,
" seeing that, by the help of that *limit*, the computation would
" be rendered practicable in the case wherein, without such
" help, the before-mentioned theorems fail. The result of
" my endeavours respecting that point appears in this Me-
" moir: which, among other matters, contains the investiga-
" tion of a general theorem for finding the length of any arc
" of any conic hyperbola by means of two elliptic arcs."—
Vol. I. p. 23 and 24.

And towards the end of the same memoir he has expressed himself thus: " Mr. MACLAURIN's method of construction," [of the elastic curve,] " just now adverted to, though very " elegant, is not without a defect. The difference between the " hyperbolic arc and its tangent being necessary to be taken, " the method (for the reason mentioned at the beginning " of this Memoir) always fails when some principal point in " the figure is to be determined; the said arc and its tangent " then both becoming infinite, though their difference be at " the same time finite." P. 36.

3. Whoever reads the passages here quoted, and knows not what was done on the subject before Mr. LANDEN handled it, will undoubtedly conceive that he was the first person who solved the problem *of computing the difference between the length of the infinite arch of an hyperbola and its asymptote*. Yet the fact is not so. That difference may be computed, in many cases, by the first series given by Mr. MACLAURIN in Art. 808 of his *Treatise of Fluxions*, which series admits of an easy transformation into another form, by which the aforesaid difference may be computed in all cases; or the fluent may be

taken in a series which will always converge; and both he and Mr. SIMPSON have actually produced such a series, the one in the place before referred to, and the other in Art. 435 of his *Doctrine of Fluxions*. And although this series, when the transverse axis of the hyperbola is much greater than the conjugate, will converge very slowly, yet (as I have shown in the Philosophical Transactions for the year 1798,) the value of it to seven, or even to ten places of figures is, in all cases, attainable.

As Mr. LANDEN had the character of a man of great probity, and as he has, in various parts of his writings, shewn a regard for the memory of the eminent mathematicians above-mentioned, I cannot account for this misrepresentation of the matter any other way than by supposing that, being old when he wrote this memoir, and incumbered with much other business, his memory failed him. His just praise on this occasion is, that his solution of the aforesaid problem is much better than those of his contemporaries. I shall have occasion to speak again of this problem in my observations towards the end of this paper; but now proceed to the proof of the main point which I had in view in writing it.

4. If the transverse axis of any conic hyperbola be called 2, and the conjugate axis $2b$; and if the abscissa counted from the center on the transverse axis be called x, the corresponding ordinate y, and the length of the arch from the vertex to the ordinate H; then, by SIMPSON's *Fluxions*, Art. 435, we have $\dot{H} = \frac{\dot{x}\sqrt{(xx + bbxx - 1)}}{\sqrt{(xx-1)}}$, or, (putting $ee = 1 + bb$,) $\dot{H} = \frac{\dot{x}\sqrt{(eexx-1)}}{\sqrt{(xx-1)}}$. [1]

5. If now we put $x = \surd\left(\frac{1-\frac{nn}{ee}}{1-nn}\right)$, [2] we shall have $xx = \frac{1-\frac{nn}{ee}}{1-nn}$, $eexx = \frac{ee-nn}{1-nn}$, and $eexx - 1 = \frac{ee-nn}{1-nn} - 1 = \frac{ee-nn-1+nn}{1-nn} = \frac{ee-1}{1-nn}$; we shall also have $xx - 1 = \frac{1-\frac{nn}{ee}}{1-nn} - 1 = \frac{1-\frac{nn}{ee}-1+nn}{1-nn} = \frac{\left(1-\frac{1}{ee}\right)nn}{1-nn}$; and thence, by substitution, $\frac{eexx-1}{xx-1} = \frac{ee-1}{1-nn} \div \frac{\left(1-\frac{1}{ee}\right)nn}{1-nn} = \frac{e^4-ee}{(ee-1)nn} = \frac{ee}{nn}$, and consequently $\surd\left(\frac{eexx-1}{xx-1}\right) = \frac{e}{n}$. [3]

But, since x was put $= \surd\left(\frac{1-\frac{nn}{ee}}{1-nn}\right)$, (see the equation numbered [2], we have $\dot{x} = \frac{-\frac{\dot{n}n}{ee}}{\surd\left(1-\frac{nn}{ee}\right)} \times \frac{1}{\surd(1-nn)} + \frac{\dot{n}n\surd\left(1-\frac{nn}{ee}\right)}{(1-nn)^{\frac{3}{2}}}$, which, by reduction, becomes $\frac{\left(1-\frac{1}{ee}\right)\dot{n}n}{(1-nn)^{\frac{3}{2}} \times \surd\left(1-\frac{nn}{ee}\right)}$. And lastly, by substituting this value of $\dot{x}$ in the equation numbered [3], we have $\dot{H} = \dot{x}\surd\left(\frac{eexx-1}{xx-1}\right) = \frac{\left(1-\frac{1}{ee}\right)\dot{n}n}{(1-nn)^{\frac{3}{2}} \times \surd\left(1-\frac{nn}{ee}\right)} \times \frac{e}{n} = \frac{e\left(1-\frac{1}{ee}\right)\dot{n}}{(1-nn)^{\frac{1}{2}} \times \surd\left(1-\frac{nn}{ee}\right)}$,* the fluxion of the arch of the hyperbola.

6. Let us now (to simplify the expression) put $e = \frac{1}{e}$, and

* As this result differs from that given by Mr. WOODHOUSE, in p. 260 of the Philosophical Transactions for 1804, I have set down the process at large, that the intelligent reader may the more easily perceive where the truth lies.

assume $V = u\sqrt{\left(\frac{1-ss\,uu}{1-uu}\right)}$; then, by taking the fluxions on both sides, we shall have $\dot{V} = \dot{u}\sqrt{\left(\frac{1-ss\,uu}{1-uu}\right)} - \frac{ss\,\dot{u}\,uu}{\sqrt{(1-uu)\times\sqrt{(1-ss\,uu)}}} + \frac{\dot{u}\,uu\sqrt{(1-ss\,uu)}}{(1-uu)^{\frac{3}{2}}}$. Here $\dot{u}\sqrt{\left(\frac{1-ss\,uu}{1-uu}\right)}$, the first term on the right hand side of the equation, is evidently the fluxion of an arch of an ellipsis of which the transverse semi-axis is 1, the eccentricity is s, and the abscissa (counted from the center) is u. The third term, $\frac{\dot{u}\,uu\sqrt{(1-ss\,uu)}}{(1-uu)^{\frac{3}{2}}}$, by multiplying both numerator and denominator by $\sqrt{(1-ss\,uu)}$, becomes $\frac{\dot{u}\,uu-ss\,\dot{u}\,u^4}{(1-uu)^{\frac{3}{2}}\times\sqrt{(1-ss\,uu)}}$, which (by division) is very easily resolved into $\frac{ss\,\dot{u}\,uu}{\sqrt{(1-uu)}\times\sqrt{(1-ss\,uu)}} + \frac{(1-ss)\dot{u}\,uu}{(1-uu)^{\frac{3}{2}}\times\sqrt{(1-ss\,uu)}}$; so that the sum of the second and third terms on that side of the equation becomes barely $\frac{(1-ss)\dot{u}\,uu}{(1-uu)^{\frac{3}{2}}\times\sqrt{(1-ss\,uu)}}$, which expression is obviously $= -\frac{(1-ss)\dot{u}}{\sqrt{(1-uu)}\times\sqrt{(1-ss\,uu)}} + \frac{(1-ss)\dot{u}}{(1-uu)^{\frac{1}{2}}\times(1-ss\,uu)}$; and the last of these (s being by the notation $= \frac{1}{e}$) differs from $\dot{H}$, the fluxion of the hyperbolic arch, found in the preceding Article, only in that it is not multiplied by e; or, in other words, it is $= \frac{\dot{H}}{e}$. Let us therefore substitute the values now found for their equals in the above fluxional equation, and we shall have $\dot{V} = \dot{u}\sqrt{\left(\frac{1-ss\,uu}{1-uu}\right)} - \frac{(1-ss)\dot{u}}{\sqrt{(1-uu)}\times\sqrt{(1-ss\,uu)}} + \frac{\dot{H}}{e}$. (*a*)

7. Now it is easy to perceive, from what is done in Art. 13 and 14 of LANDEN's second *Memoir*, (above referred to,) that the fluent of the second term on the right-hand side of this equation may be found by means of the fluent of the first term,

an algebraic quantity, and the arch of another ellipsis which is more eccentric. And as this has been done by some late writers on the rectification of the ellipsis,* I shall, on the present occasion, only state and use the result, in a notation convenient for an arithmetical calculator. Thus, putting $\zeta = \frac{2\sqrt{e}}{1+e}$, and $vv = \frac{e+uu-\sqrt{(ee-uu)(1-uu)}}{2e}$,

and the fluent of $\frac{\dot{u}\sqrt{(1-n\,uu)}}{\sqrt{(1-uu)}} = \mathrm{E}$,

\- - - of $\frac{\dot{u}}{\sqrt{(1-uu)}\times\sqrt{(1-n\,uu)}} = \mathrm{F}$,

\- - - of $\frac{\dot{v}\sqrt{(1-\zeta\zeta\,vv)}}{\sqrt{(1-vv)}} = \mathrm{'E}$, it is certain that

$$\mathrm{F\ is} = \frac{\zeta\zeta}{2}\left(\frac{1+e}{1-e}\right)u + \frac{2}{1-n}\,\mathrm{E} - \frac{2}{1-e}\,\mathrm{'E}.$$

Multiply this equation by $1-ee$, and write $2e$ for its equal $\frac{\zeta\zeta}{2}(1+e)^2$, and it becomes

$$(1-ee)\,\mathrm{F} = 2e\,u + 2\mathrm{E} - 2(1+e)\,\mathrm{'E}.$$

If we now take the fluents of the fluxions in the equation marked (α) in the preceding Article, we shall have $\mathrm{V} = \mathrm{E} - (1-ee)\,\mathrm{F} + \frac{\mathrm{H}}{e}$. And since all these quantities begin and increase together, this equation needs no correction. And by writing for $(1-ee)\,\mathrm{F}$ its value found in the preceding equation, we have $\mathrm{V} = -2e\,u - \mathrm{E} + 2(1+e)\,\mathrm{'E} + \frac{\mathrm{H}}{e}$; and thence by reduction and transposition, $\mathrm{H} = 2ee\,u + e\,\mathrm{V} + e\,\mathrm{E} - 2e(1+e)\,\mathrm{E}'$; which expression will be found to exceed that given (for the same purpose) by Mr. WOODHOUSE, in p. 261 of the Philosophical Transactions for the year 1804, in the ratio of e to 1.

* See M. LACROIX's *Traité du Calcul Différentiel et du Calcul Intégral*, Tome II. p. 181.

Moreover; since by the notation in Art. 5 and 6, $e\,e$ is $=1$, and $V=ux$, we have, by substituting these values for their equals in the last equation, $H=2u+e\,(ux+E)-2\,(1+e)$ E, (ζ) which is LANDEN's theorem in a different notation.

8. I am now to prove, that all the labour of computing the eccentricity* and abscissa, and the arch itself, of this second ellipsis, and the subsequent operations of multiplication and subtraction requisite in the application of it to the rectification of the hyperbola, is altogether needless; since the same end may be obtained by only computing and applying the fluent of $\dot{u}\sqrt{\left(\frac{1-nn}{1-u\,uu}\right)}$, which will require no more calculation than must be made to find and apply the elliptic arch denoted by E (of which the axis, and eccentricity, and abscissa, are given). And the truth of this will quickly appear. For,

9. The fractional expression, $\frac{\dot{u}\sqrt{(1-u\,uu)}}{\sqrt{(1-uu)}}-\frac{(1-u)\,\dot{u}}{\sqrt{(1-uu)}\sqrt{(1-u\,uu)}}$, found above, in the equation marked (α), (see Art. 6.) by reduction to a common denominator, becomes

$$\frac{\dot{u}-u\,uu\,\dot{u}-\dot{u}+u\,\dot{u}}{\sqrt{(1-uu)}\times\sqrt{(1-u\,uu)}}=\frac{u\,\dot{u}(1-uu)}{\sqrt{(1-uu)}\times\sqrt{(1-u\,uu)}}=\frac{u\,\dot{u}\sqrt{(1-uu)}}{\sqrt{(1-u\,uu)}}.$$

Substitute this for its equal in the aforesaid equation, and we have $\dot{V}=\frac{u\,\dot{u}\sqrt{(1-uu)}}{\sqrt{(1-u\,uu)}}+\frac{\dot{H}}{e}$. (γ). Take the fluents, denoting that of $\frac{\dot{u}\sqrt{(1-uu)}}{\sqrt{(1-u\,uu)}}$ by G, and there will be $V=u\,G+\frac{H}{e}$; and thence, by transposition and multiplication, $H=e\,V-e\,G$. (δ). This theorem, as far as I know, is new.

Here then it appears, that the rectification of the hyperbola is accomplished by means of the algebraic quantity $e\,V$,

* When it is more convenient to use the conjugate semi-axis than the eccentricity, in the arithmetical calculation, then that must be computed instead of the eccentricity.

$(= eu\sqrt{\left(\frac{1-ss\,xx}{1-xx}\right)} = eux,)$ and e x the fluent of $\frac{\dot{s}\sqrt{(1-ss\,xx)}}{\sqrt{(1-ss\,xx)}}$. And it is obvious to every competent judge that the arithmetical work of computing the value of G, with any given values of s and u, will be as short and easy as the computation of the elliptic arch denoted by E. Yet, for the more ready comparison of the series, with each other, which arise in taking these fluents, I will here set them down in the NEWTONIAN form, which undoubtedly is the most convenient for arithmetical calculations that has yet been discovered.

10. $\dot{E}$ is $= \frac{\dot{s}\sqrt{(1-ss\,uu)}}{\sqrt{(1-uu)}} = \frac{\dot{s}}{\sqrt{(1-uu)}} \times : 1 - \frac{ss\,uu}{2} - \frac{s^4u^4}{2.4} - \frac{3.s^6u^6}{2.4.6} - \frac{3.5.s^8u^8}{2.4.6.8}$, &c.; and denoting the fluents of $\frac{\dot{s}}{\sqrt{(1-uu)}}$, $\frac{\dot{s}\,uu}{\sqrt{(1-uu)}}$, $\frac{\dot{s}\,u^4}{\sqrt{(1-uu)}}$, &c. by A′, B′, C′, &c. respectively, we have

$A' =$ the circ. arch of which the rad. is 1, and sine u.

$B' = \frac{A'-u\sqrt{(1-uu)}}{2}$,

$C' = \frac{3B'-u^3\sqrt{(1-uu)}}{4}$,

$D' = \frac{5C'-u^4\sqrt{(1-uu)}}{6}$,

$E' = \frac{7D'-u^7\sqrt{(1-uu)}}{8}$,

&c. &c. And then, multiplying these quantities by their proper factors, and placing them in due order, we have

$$E = A' - \frac{ss}{2}B' - \frac{s^4}{2.4}C' - \frac{3.s^6}{2.4.6}D' - \frac{3.5.s^8}{2.4.6.8}E', \text{ &c.}$$

11. $\dot{G}$ is $= \frac{\dot{s}\sqrt{(1-ss\,uu)}}{\sqrt{(1-ss\,xx)}} = \dot{u}\sqrt{(1-uu)} \times : 1 + \frac{ss\,uu}{2} + \frac{3.s^4u^4}{2.4} + \frac{3.5.s^6u^6}{2.4.6} + \frac{3.5.7.s^8u^8}{2.4.6.8}$, &c. Now, denoting the fluents of $\dot{u}\sqrt{(1-uu)}$, $\dot{u}\,uu\sqrt{(1-uu)}$, $\dot{u}\,u^4\sqrt{(1-uu)}$, &c. by A, B, C, &c. respectively, we have

A = area of $\frac{1}{2}$ the mid. zone of a circ. of which the rad. is 1, and sine u.

$B = \frac{A - u(1-uu)^{\frac{1}{2}}}{4}$,

$C = \frac{3B - u^3(1-uu)^{\frac{1}{2}}}{6}$,

$D = \frac{5C - u^5(1-uu)^{\frac{1}{2}}}{8}$,

$E = \frac{7D - u^7(1-uu)^{\frac{1}{2}}}{10}$,

&c. &c. And, multiplying these quantities by their proper factors, and placing them in due order, we have

$$G = A + \frac{u}{2}B + \frac{3e^2}{2.4}C + \frac{3.5e^4}{2.4.6}D + \frac{3.5.7e^6}{2.4.6.8}E, \text{ \&c.}$$

12. Now, by comparing together the fluents denoted by A′ and A, B′ and B, C′ and C, &c. it is obvious that the arithmetical calculation of the one will not, in any respect, be more difficult than that of the other; and that A is always less than A′, B than B′, C than C′, &c. And it is evident that each of the series denoted by E and G converges by the same geometrical progression, viz. e^2, e^4, e^6, &c. So that the arithmetical value of any number of terms of the latter series will always be nearer to the value of the whole, than the arithmetical value of the same number of terms of the former series will be to its whole. And as to transformations of the expression $\dot{u}\sqrt{\left(\frac{1-nuu}{1-uu}\right)}$ into others, in order to obtain the fluent in series of swifter convergency, when the case requires it; it is obvious that similar operations may be performed on the expression $\dot{u}\sqrt{\left(\frac{1-uu}{1-nuu}\right)}$.

In the application of E and G to the rectification of the hyperbola, the one is multiplied by e, the other by ϵ, (see the

equations marked (ε) and (δ) in Art. 7 and 9,) which are operations equally easy.

Thus it appears, that all the labour of computing the eccentricity, the abscissa, and the length of the elliptic arch denoted by E, and of applying it to the rectification of the hyperbola, is wholly unnecessary; and consequently, that *that method is circuitous, and more curious than useful.**

Some further Observations on the Rectification of the Hyperbola: among which the great Advantage of descending Series over ascending Series, in many cases, is clearly shown; and several Methods are given for computing the constant Quantity by which those Series differ from each other.

13. The new series above given in Art. 11, it is obvious, will converge swiftly so long as u is small in comparison of 1; (which it will be when x is not much greater than 1;) so that this series will be very convenient for computing a small arch of an hyperbola near the vertex, even when e is nearly $= 1$: and, when e is small in comparison of 1, any arch, how great so ever, may easily be computed by it. But, when e is nearly $= 1$, and uu is greater than $\frac{1}{4}$, this series will converge but slowly; and, for that reason, will not be an eligible form for arithmetical calculation. In such cases, however, a swift convergency will take place in some of the descending series (discovered by me, and) inserted in the Philosophical Trans-

* By comparing the expressions marked γ and δ, in Article 9 of this Paper, with the short paragraph near the top of page 261 of the Philosophical Transactions for 1804, the mathematical reader will quickly perceive that Mr. WOODHOUSE has there asserted too much.

actions for 1802. And as those descending series differ from the ascending ones by a constant quantity, (as is there shown,) I will now add something to what was then said on the methods of computing the value of that constant quantity.

14. It is very evident from what is done in the Philosophical Transactions for 1802, from page 460 to page 465, that the constant quantity here spoken of is no other than the difference between the length of the arch of the hyperbola and its tangent, "when the point of contact" (to use LANDEN's words,) "is supposed to be carried to an infinite distance from "the vertex of the curve:" *—which difference is undoubtedly the same as "the difference between the length of the arch "infinitely produced and its asymptote"—(as SIMPSON† expresses it.) And since each of these eminent mathematicians, and MAC LAURIN also, (as I before observed,) has treated of this difference, it seems requisite that I should here give a brief statement of their methods of computing it, and compare them with such of my own as I offer to the public.

15. If 1 be written instead of a, in Art. 435 of SIMPSON's Fluxions, the eccentricity will be $\sqrt{1+bb}$, which is denoted by e in this Paper; and his $dd = \frac{1}{1+bb}$ will be $= \frac{1}{ee} = ii$. Substituting, therefore, 1 for a, and i for d, in the series by which he expresses the difference between the length of the asymptote and the infinite arch, it becomes

$$\mathrm{A}\,x : \tfrac{i}{2} + \tfrac{i^3}{2.2.4} + \tfrac{3.3i^5}{2.2.4.4.6} + \tfrac{3.3.5.5i^7}{2.2.4.4.6.6.8}, \text{ \&c.}$$

his A being = the quadrantal arch of a circle of which the radius is 1.

In like manner, 1 being written instead of a, in the second

* See LANDEN's Memoirs, Vol. I. p. 23. † See his Fluxions, Art. 435.

series in Art. 808 of MAC LAURIN's Fluxions, his $E = a + \frac{bb}{a}$ will become $= 1 + bb$, which is $= ee$ in the notation used in this paper; and his series,

$$\frac{Na}{2} \times \sqrt{\frac{a}{E}} + \frac{aA}{2.4E} + \frac{9aB}{4.6E} + \frac{25aC}{6.8E} + \frac{49aD}{8.10E}, \text{ \&c.}$$

(A denoting the first term, B the second, C the third, &c.) will become $N \times : \frac{1}{2e} + \frac{1}{2.2.4e^3} + \frac{3.3}{2.2.4.4.6e^5} + \frac{3.3.5.5}{2.2.4.4.6.6.8e^7}$, &c. which series, since $\frac{1}{e}$ is $= \epsilon$, and N denotes the quadrantal arch of a circle of which the radius is 1, exactly agrees with SIMPSON's series.

And this series will be found to agree also with the value of ϵG in the equation marked (δ) in Art. 9, when u becomes $= 1$. For, in that case, $eV = ex$ denotes the asymptote, and H the infinite arch of the hyperbola; and we have, by transposition, $ex - H = \epsilon G$. And, u being $= 1$, A, the first term of the series denoted by G in Art. 11, becomes $=$ the area of a quadrant of a circle of which the radius is 1, that is, $= \frac{N}{2}$: B becomes $= \frac{A}{4} = \frac{N}{2.4}$: C becomes $= \frac{3B}{6} = \frac{3N}{2.4.6}$: D becomes $= \frac{5C}{8} = \frac{3.5N}{2.4.6.8}$: &c. and these values being written for A, B, C, D, &c. and the whole $\times \epsilon$, we have $\epsilon G = N \times : \frac{\epsilon}{2} + \frac{\epsilon^3}{2.2.4} + \frac{3.3\epsilon^5}{2.2.4.4.6} + \frac{3.3.5.5\epsilon^7}{2.2.4.4.6.6.8}$, &c. perfectly agreeing with the series above stated.

This series, it is obvious, will converge but slowly when e is not much greater, and consequently ϵ not much less, than 1; that is when b is small in comparison of 1. But, in such cases, other series which have a good rate of convergency may be used, as was shown in my former paper on the Rectification

of the Hyperbola, and will more fully appear in the following pages.

Mr. LANDEN's methods of computing the aforesaid difference next come under consideration: and, first, his method of computing it by means of the arches of two ellipses.

16. We have seen above, that, when x becomes immensely great, u becomes $= 1$; and in this case the equation (C) in Art. 7 becomes $H = 2 + ex + eE - 2(1+e)'E$; from which we have $ex - H = 2(1+e)'E - eE - 2$, another expression of the difference between the length of the asymptote and the infinite arch: which expression, however, is not so convenient for arithmetical calculation as the preceding. For here E denotes the quadrantal arch of an ellipsis of which the transverse semi-axis is 1, and the eccentricity e; which arch is $=$

$$N \times : 1 - \frac{e}{2.2} - \frac{3e^2}{2.2.4.4} - \frac{3.3.5e^3}{2.2.4.4.6.6} - \frac{3.3.5.5.7e^4}{2.2.4.4.6.6.8.8}, \text{ \&c.}$$

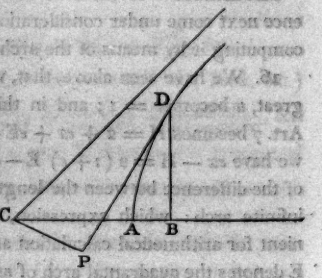
so that the computation of $eE = \frac{E}{e} =$

$$N \times : e - \frac{e}{2.2} - \frac{3e^2}{2.2.4.4} - \frac{3.3.5e^3}{2.2.4.4.6.6} - \frac{3.3.5.5.7e^4}{2.2.4.4.6.6.8.8}, \text{ \&c.}$$

will require as much labour as the computation of eG, which is the very difference sought. (See the preceding Art.) But, by this method, we have yet to compute the elliptic arch denoted by 'E, of which the transverse semi-axis is 1, the eccentricity $\zeta = \frac{2\sqrt{e}}{1+e}, \left(= \frac{2\sqrt{e}}{1+e}\right)$ and abscissa $v = \sqrt{\left(\frac{1+1}{2}\right)}, \left(= \sqrt{\left(\frac{e+1}{2e}\right)}\right)$; and then there must be a multiplication of this quantity by $2(1+e)$, and, after that, a subtraction of $eE + 2$ from the product, to obtain the difference sought: all which labour is more than is required by the method described in the preceding article. Here, then, we have a striking instance (and a thousand more might be produced) of the inutility of rectifying the hyperbola by means of two ellipses.

17. But Mr. LANDEN discovered another and a better method of computing the difference in question, which is briefly this:

If from C, the center of an hyperbola, CP be drawn perpendicular to DP, a tangent to the curve in D; and if the transverse semi-axis CA be $= m$, the conjugate semi-axis $= n$, the perpendicular CP $= p$; and if f be put $= \frac{mm-nn}{2m}$, and $z = \frac{pp}{m}$;



then will the difference between the length of the tangent DP and the arch AD be universally expressed by d — the fluent of $\frac{\dot{z}\sqrt{mz}}{z\sqrt{(nn+zf-zz)}}$, where d is a constant quantity $=$ the fluent generated while z increases from o to m.

If now $\frac{pp}{m}$ be written instead of z, the above fluxion will become $\frac{\dot{p}\,pp}{\sqrt{(m^2n^2+2fm-p^4)}}$, agreeing with MAC LAURIN's expression of the same thing in Art. 804 and 808 of his Fluxions, where the transverse and conjugate semi-axes are denoted by a and b, respectively; and, in the latter of those articles, this expression is resolved into $\frac{\dot{p}\,pp}{\sqrt{(aa-pp)}\sqrt{(bb+pp)}}$, and its fluent generated while p increases from o to a, or m, is exhibited in series of which I have spoken in Art. 8 and 15 of this Paper.

Mr. LANDEN has no where, that I know of, exhibited the fluent of the above fluxion in series, but in the following manner.* Denoting the fluent of $\frac{\dot{p}\,pp}{\sqrt{(mm-pp)}\sqrt{(nn+pp)}}$, generated

* See his 2d Memoir, Art. 5.

while p increases from o to m, by L, (which I denote by d,) he says that, when the abscissa CB is $= m \times \surd\left(1 + \frac{n}{\surd(mm+nn)}\right)$, (at which time the ordinate BD is $= n \times \surd\left(\frac{n}{\surd(mm+nn)}\right)$, and the tangent DP $= \surd(mm+nn)$), then L, (or d,) is $=$ 2 DP $-$ 2 AD $+ n - \surd(mm+nn) = n + \surd(mm+nn) -$ 2 AD.

18. Now, in order to compare this method with those of Mac Laurin and Simpson which have been described above, we may proceed thus: $\frac{\dot{p}\,pp}{\surd(mm-pp)\,\surd(nn+pp)}$ (which, for the sake of brevity in a subsequent use of the fluent, I denote by $\dot{\phi}$,) is $= \frac{\dot{p}\,pp}{m\surd(nn+pp)} \times : 1 + \frac{pp}{2mm} + \frac{3p^4}{2.4m^4} + \frac{3.5p^6}{2.4.6m^6} + \frac{3.5.7p^8}{2.4.6.8m^8}$, &c.; it is also

$= \frac{\dot{p}\,pp}{n\surd(mm-pp)} \times : 1 - \frac{pp}{2nn} + \frac{3p^4}{2.4n^4} - \frac{3.5p^6}{2.4.6n^6} + \frac{3.5.7p^8}{2.4.6.8n^8}$, &c.; the one series proceeding by the powers of $\frac{pp}{mm}$, the other by the powers of $\frac{pp}{nn}$; which geometrical progressions, assisted by numeral coefficients, it is obvious, will have place also in the fluents as they are exhibited here below. Thus,

The fluents of $\frac{\dot{p}\,pp}{\surd(nn+pp)}$, $\frac{\dot{p}\,p^4}{\surd(nn+pp)}$, $\frac{\dot{p}\,p^6}{\surd(nn+pp)}$, &c. being denoted by A, B, C, &c. respectively, we shall have

$$A = \frac{p\surd(nn+pp) - nn \text{ H. L.}\left(\frac{p}{n} + \surd\left(1+\frac{pp}{nn}\right)\right)}{2},$$

$$B = \frac{p^3\surd(nn+pp) - 3nn\,A}{4},$$

$$C = \frac{p^5\surd(nn+pp) - 5nn\,B}{6},$$

$$D = \frac{p^7\surd(nn+pp) - 7nn\,C}{8},$$

$$E = \frac{p^9\surd(nn+pp) - 9nn\,D}{10},$$

&c. &c.

and hence,

$1^{\circ}.\ \varphi = \frac{1}{m} \times : A + \frac{1}{2mm} B + \frac{3}{2.4m^4} C + \frac{3.5}{2.4.6m^6} D + \frac{3.5.7}{2.4.6.8m^8} E$, &c.

And the fluents of $\frac{\dot{p}\,pp}{\sqrt{(mm-pp)}}$, $\frac{\dot{p}\,p^4}{\sqrt{(mm-pp)}}$, $\frac{\dot{p}\,p^6}{\sqrt{(mm-pp)}}$, &c. being denoted by A′, B′, C′, &c. respectively, we shall have

$$A' = \frac{mm \times \text{cir. arch of which rad. is 1 and sine } \frac{p}{m} - p\sqrt{(mm-pp)}}{2},$$

$$B' = \frac{3mmA' - p^3\sqrt{(mm-pp)}}{4},$$

$$C' = \frac{5mmB' - p^5\sqrt{(mm-pp)}}{6},$$

$$D' = \frac{7mmC' - p^7\sqrt{(mm-pp)}}{8},$$

$$E' = \frac{9mmD' - p^9\sqrt{(mm-pp)}}{10},$$

&c. &c.

and hence,

$2^{\circ}.\ \varphi = \frac{1}{n} \times : A' - \frac{1}{2nn} B' + \frac{3}{2.4n^4} C' - \frac{3.5}{2.4.6n^6} D' + \frac{3.5.7}{2.4.6.8n^8} E'$, &c.

Each of these series begins and increases with p; so that neither of them needs any correction.

19. Two series being thus obtained as expressions of the general value of the fluent of $\frac{\dot{p}\,pp}{\sqrt{(mm-pp)}\sqrt{(nn+pp)}}$, (denoted by φ,) the next thing to be done is, to ascertain the rate of convergency of each when the abscissa CB is $= m\sqrt{\left(1 + \frac{n}{(mm+nn)}\right)}$. Now to this value of the abscissa CB, the corresponding value of the perpendicular CP $= p$ (as appears from the equations in Art. 17,) is $= m\sqrt{\left(\frac{n}{n+\sqrt{(mm+nn)}}\right)}$. We therefore have, in this case, $\frac{pp}{mm} = \frac{n}{n+\sqrt{(mm+nn)}}$, and $\frac{pp}{nn} = \frac{mm}{nn+n\sqrt{(mm+nn)}}$, which are the respective rates of convergency of the geometrical pro-

gressions which (assisted by numeral coefficients) will have place in the first and second series given in the preceding article.

20. It is now easy to compare these rates of convergency with that which has place in the series in Art. 15, thus: Putting 1 instead of m, and b instead of n, the rates are these, viz.

In 1st Series.	In 2d Series.	In Art. 15.
$\frac{b}{b+\sqrt{(1+bb)}}$,	$\frac{1}{bb+b\sqrt{(1+bb)}}$,	$\frac{1}{1+bb}$.

And writing 1, 2, 3, &c. successively instead of b, we have

$\frac{1}{1+\sqrt{2}}$,	$\frac{1}{1+\sqrt{2}}$,	$\frac{1}{2}$,
$\frac{2}{2+\sqrt{5}}$,	$\frac{1}{4+2\sqrt{5}}$,	$\frac{1}{5}$,
$\frac{3}{3+\sqrt{10}}$,	$\frac{1}{9+3\sqrt{10}}$,	$\frac{1}{10}$,
&c.	&c.	&c.

In all which cases it is evident, that the calculation by the series given in Art. 15 will, on a double account of simplicity, be much easier than by the second series in the preceding Art. notwithstanding its greater rate of convergency.

Let us now write $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. successively instead of b, and we shall have the following rates

In 1st Series.	In 2d Series.	In Art. 15.
$\frac{1}{1+\sqrt{5}}$,	$\frac{4}{1+\sqrt{5}}$,	$\frac{4}{5}$,
$\frac{1}{1+\sqrt{10}}$,	$\frac{9}{1\sqrt{+10}}$,	$\frac{9}{10}$,
$\frac{1}{1+\sqrt{17}}$,	$\frac{16}{1+\sqrt{17}}$,	$\frac{16}{17}$,
&c.	&c.	&c.

Here the great advantage of the first series given in the

preceding Art. over that which was discovered by MACLAURIN and SIMPSON appears; and we see that LANDEN had good grounds for valuing his method, or, to express myself better, one of his methods of solving the Problem which I have now under consideration, although it cannot be truly said that he finished his work.

21. If indeed the hyperbola were equilateral, then, n being $= m$, the fluxional expression $\frac{\dot{p}\,pp}{\sqrt{(mm-pp)}\sqrt{(nn+pp)}}$ becomes $= \frac{\dot{p}\,pp}{\sqrt{(m^4-p^4)}} = \frac{\dot{p}\,pp}{mm} \times : 1 + \frac{p^4}{2m^4} + \frac{3p^8}{2.4m^8} + \frac{3.5p^{12}}{2.4.6m^{12}}$, &c.; and we have

$$\varphi = \frac{p^3}{mm} \times : \frac{1}{3} + \frac{p^4}{2.7m^4} + \frac{3p^8}{2.4.11m^8} + \frac{3.5p^{12}}{2.4.6.15m^{12}}, \text{ \&c.}$$

And taking $p = m\sqrt{\left(\frac{n}{n+\sqrt{(mm+nn)}}\right)}$, which, in this case, is $= m\sqrt{\left(\frac{1}{1+\sqrt{2}}\right)}$, we shall have $\frac{p^4}{m^4} = \frac{m^4}{m^4}\left(\frac{1}{1+\sqrt{2}}\right)^2 = \frac{1}{3+\sqrt{8}}$, the rate of convergency of the geometrical progression which will have place in this series; and this rate of convergency, together with the simplicity of the numeral coefficients, will render this series eligible for numerical calculation in preference to either of the other series.

22. The fluent of $\frac{\dot{p}\,pp}{\sqrt{(mm-pp)}\sqrt{(nn+pp)}}$, when $p = m\sqrt{\left(\frac{n}{n+\sqrt{(mm+nn)}}\right)}$, being obtained in *converging series*, whatever be the ratio of m to n; let this particular value of it be denoted by Φ, (to distinguish it from the general value denoted by φ,) and substituted in the general equation $d - \varphi = \mathrm{DP} - \mathrm{AD}$, in Art. 17, and we shall have $d - \Phi = \mathrm{DP} - \mathrm{AD}$; and this value of $\mathrm{DP} - \mathrm{AD}$ being substituted for it in the particular equation $\mathrm{L} = d = 2\mathrm{DP} - 2\mathrm{AD} + n - \sqrt{(mm+nn)}$, in the same

Article (in which equation DP $= \sqrt{(mm+nn)}$,) we have $d = 2d - 2\Phi + n - \sqrt{(mm+nn)}$, and hence $L = d = 2\Phi + \sqrt{(mm+nn)} - n$, which is the difference, or quantity sought.

23. I come now to make a comparison of Mr. LANDEN's last mentioned method of computing the difference in question with some of my own methods.

We have already seen in Art. 20, that, when the conjugate axis of the hyperbola is greater than the transverse, LANDEN's method is not wanted, since the operation by the old series will, in general, be easier. The comparison therefore, now to be made, is only in cases when the conjugate axis is equal to, or less than, the transverse axis.

It appears from Art. 9, 11, and 17 of this Paper, that, when m is put $= 1$, and x is taken $= m\sqrt{\left(1 + \frac{n}{\sqrt{(mm+nn)}}\right)}$, the value of $ss\,uu$ is $\frac{1}{1+nn+n\sqrt{(1+nn)}}$, the powers of which fraction form the geometrical progression which will have place in the series denoted by G, from which H, or the arch AD is quickly obtained. And, with these values of m and x, we have, by Art. 19, $\frac{pp}{mm} = \frac{n}{n+\sqrt{(1+nn)}}$, the powers of which fraction form the geometrical progression which will have place in the series denoted by Φ. When n is taken $= 1$, the former of these algebraic fractions is $= \frac{1}{2+\sqrt{2}}$, the latter is $= \frac{1}{1+\sqrt{2}}$. As the one fraction increases with n, and the other decreases, it is easy to find that value of n which shall make them equal, viz. $n = \frac{1}{2}\sqrt{(-2+\sqrt{20})} = 0{\cdot}786$, &c.; and, with this value of n, each of these fractions is $= \frac{3-\sqrt{5}}{2} = 0{\cdot}381966$, &c. Thus it appears, that, n having any value greater than $\frac{1}{2}\sqrt{(-2+}$

$\sqrt{20}$), the fraction $\frac{1}{1+nn+n\sqrt{(1+nn)}}$ will be less than $\frac{n}{n+\sqrt{(1+nn)}}$, and less than 0·881966; and consequently that the hyperbolic arch AD may be more easily obtained from the series denoted by G, in Art. 11, than by either of those denoted by ϕ in Art. 18. And hence we may derive another expresssion of the value of d, in the following manner.

24. Since uu is universally $= \frac{ee\,xx-ee}{ee\,xx-1}$, (see equation [3] in Art. 5, where e denotes the eccentricity;) by putting $xx = 1 + \frac{n}{\sqrt{(1+nn)}}$, and writing $1+nn$ instead of ee, we shall have $uu = \frac{\sqrt{(1+nn)}}{n+\sqrt{(1+nn)}}$, and $ee\,uu = \frac{uu}{ee} = \frac{1}{1+nn+n\sqrt{(1+nn)}}$. Let the value of G, corresponding to these values of u and e, be denoted by Γ; then, by the equation (δ) in Art. 9, the hyperbolic arch AD $=$ H, is $= eV - e\Gamma$. But, since V was put $= ux$, (see Art. 5 and 6,) it will in this case be $= 1$. Writing therefore $e - e\Gamma$ for AD, and 1 for m, in the equation at the end of Art. 17, we have $d = n + \sqrt{(1+nn)} - 2e + 2e\Gamma = 2e\Gamma + n - \sqrt{(1+nn)}$. [$\eta$]

It now appears, that d, the difference between the length of an infinite arch of an hyperbola and its tangent, or asymptote, may be computed by means of one series converging swifter than the powers of $\frac{1}{10}$, even in the most disadvantageous case; so that a dozen terms of it will be sufficient for all common uses: but, that a series of such convergency was attainable in this case, appears not to have been observed by either of the writers before mentioned.

25. If the transverse and conjugate semi-axes of an hyperbola are denoted by a and 1, respectively, the ordinate by y, and the arch by z; and if $\sqrt{(aa+1)}$, the eccentricity, be

put $= e$, and if $\frac{1}{\sqrt{y}}$ be written instead of y, in Theorems H. and IV. in pages 453 and 454 of the Philos. Trans. for 1809; then, (as appears by pages 466 and 467 of the same Vol.) the corresponding value of z may be obtained from either of the following equations, viz.

1. $z = A - \frac{1}{2}B + \frac{3}{2.4}C - \frac{3.5}{2.4.6}D + \frac{3.5.7}{2.4.6.8}E$, &c.

2. $z + d = \begin{cases} e\sqrt{(\frac{1}{e}+1)} \\ +\frac{1}{2e}A' - \frac{1}{2.4e^2}B' + \frac{3}{2.4.6e^3}C' - \frac{3.5}{2.4.6.8e^4}D' \text{, \&c.} \end{cases}$

in the first of which equations

$$A = \frac{1}{2\sqrt{e}}\sqrt{(1+e)} + \frac{1}{2e}\text{H.L.}\,(\sqrt{e} + \sqrt{(1+e)}),$$

$$B = \frac{e^{-\frac{1}{2}}(1+e)^{\frac{3}{2}} - A}{4e},$$

$$C = \frac{e^{-\frac{1}{2}}(1+e)^{\frac{3}{2}} - 3B}{6e},$$

$$D = \frac{e^{-\frac{1}{2}}(1+e)^{\frac{3}{2}} - 5C}{8e},$$

$$E = \frac{e^{-\frac{1}{2}}(1+e)^{\frac{3}{2}} - 7D}{10e},$$

&c. &c.

in the second,

$$A' = \text{H.L.}\,(\sqrt{1+e}) - \sqrt{e}),$$

$$B' = \frac{-e\sqrt{(\frac{1}{e}+1)} - A}{2},$$

$$C' = \frac{-e\sqrt{(\frac{1}{e}+1)} - 3B'}{4},$$

$$D' = \frac{-e\sqrt{(\frac{1}{e}+1)} - 5C'}{6},$$

$$E' = \frac{-e\sqrt{(\frac{1}{e}+1)} - 7D'}{8},$$

&c. &c.

Now, since $\sqrt{(1+e)} - \sqrt{e}$ is $= \frac{1}{\sqrt{(1+e)}+\sqrt{e}}$, if l be put $=$ H. L. $(\sqrt{e}+\sqrt{(1+e)})$, and if the values of A, B, C, &c. and of A′, B′, C′, &c. be taken in terms of e and l, and written for them in the above two equations, we shall have

1. $$z = \sqrt{1+e} \times \frac{1}{2e^{\frac{1}{2}}} + \frac{l}{2e}$$
$$- \sqrt{1+e} \times \left(\frac{1}{2.4e^{\frac{3}{2}}} + \frac{1}{2.2.4e^{\frac{5}{2}}}\right) + \frac{l}{2.2.4e^{2}}$$
$$+ \sqrt{1+e} \times \left(\frac{3}{2.4.6e^{\frac{5}{2}}} + \frac{3}{2.4.4.6e^{\frac{7}{2}}} - \frac{3.3}{2.2.4.4.6e^{\frac{9}{2}}}\right) + \frac{3.3l}{2.2.4.4.6e^{3}}$$
$$- \sqrt{1+e} \times \left(\frac{3.5}{2.4.6.8e^{\frac{7}{2}}} + \frac{3.5}{2.4.6.6.8e^{\frac{9}{2}}} - \frac{3.5.5}{2.4.4.6.6.8e^{\frac{11}{2}}} + \frac{3.3.5.5}{2.2.4.4.6.6.8e^{\frac{13}{2}}}\right) + \frac{3.3.5.5.5l^{2}}{2.2.4.4.6.6.8e^{5}}$$
&c. &c.

2. $$z + d = e^{\frac{1}{2}}\sqrt{1+e} - \frac{l}{2e}$$
$$+ \sqrt{1+e} \times \frac{1}{2.2.4e^{\frac{3}{2}}} - \frac{l}{2.2.4e^{2}}$$
$$- \sqrt{1+e} \times \left(\frac{3}{2.4.6e^{\frac{5}{2}}} - \frac{3.3}{2.2.4.4.6e^{\frac{7}{2}}}\right) - \frac{3.3l}{2.2.4.4.6e^{3}}$$
$$+ \sqrt{1+e} \times \left(\frac{3.5}{2.4.6.6.8e^{\frac{7}{2}}} - \frac{3.5.5}{2.4.4.6.6.8e^{\frac{9}{2}}} + \frac{3.3.5.5}{2.2.4.4.6.6.8e^{\frac{11}{2}}}\right) - \frac{3.3.5.5.5l^{2}}{2.2.4.4.6.6.8e^{5}}$$
&c. &c.

Here, on the right-hand side of both these equations, the diagonal lines of quantities in which l enters, and the perpendicular columns which have the common factor $\sqrt{1+e}$, the first column of the first equation, and the first term of the second excepted, are exactly alike, but under contrary signs; so that, by taking the sum on each side, we have

$$2z + d = \begin{cases} \sqrt{1+e} \times : \frac{1}{2e^{\frac{1}{2}}} - \frac{1}{2.4e^{\frac{3}{2}}} + \frac{3}{2.4.6e^{\frac{5}{2}}} - \frac{3.5}{2.4.6.8e^{\frac{7}{2}}} \text{ &c.} \\ + el\sqrt{1+e}. \end{cases}$$

The right-hand side of this equation, by multiplying both numerators and denominators by $e\dot{t}$, becomes $e\dot{t}\sqrt{1+e} \times : 1 + \frac{1}{2e} - \frac{1}{2.4e^2} + \frac{3}{2.4.6e^3} - \frac{3.5}{2.4.6.8e^4}$, &c. which is $= e\dot{t}\sqrt{1+e} \times \sqrt{1+\frac{1}{e}} = \sqrt{1+e} \times \sqrt{1+e} = 1+e$. Hence we have $d = 1+e-2z$, which is the very equation at the end of Art. 17, obtained by LANDEN's method, the difference being only in the notation.

It appears by this result, that the constant difference between the values of the ascending and descending series, here denoted by d, is equal to the difference between the lengths of the infinite arch and its tangent, as was observed in Art. 14, and may be briefly proved thus: by the notation specified at the beginning of this Art. and the property of the curve, as $1 : e :: y : ey$, the length of a line drawn parallel to the asymptote from the extremity of the ordinate to the transverse axis; which line, when y becomes immensely great, will coincide with the tangent drawn from the same point, and will be equal to the corresponding portion of the asymptote. And it appears by Art. 13 of my former Paper on the Rectification of the Hyperbola, (see Philos. Trans. for 1802, p. 461,) that the corresponding arch of the hyperbola, z, is $= ey - d$. We therefore have, in this case, $d =$ *the asymptote—the infinite arch.*

26. Thus is LANDEN's best theorem respecting the rectification of the hyperbola obtained by the common application of Sir ISAAC NEWTON's doctrine of infinite series. And I further observe, *in transitu*, that FAGNANI's theorem, respecting the rectification of the ellipsis, is attainable in the same easy manner. These devices are indeed very ingenious; but their

utility appears to me to be much less than has been imagined. It has been represented even in these Transactions, for the year 1804, p. 236, that FAGNANI's theorem is necessary to the investigation of EULER's series for computing the length of a quadrantal arch of an ellipsis; yet, the fact is, that FAGNANI's theorem is no more requisite on that occasion than LANDEN's theorem is in the investigation of a similar series for computing the difference between the lengths of an infinite arch of an hyperbola and its asymptote, which will be given in this Paper.

27. It appears by inspecting the values of z and $z + d$, exhibited in terms of e and l, in Art. 25, that when y becomes equal to, or greater than, $\frac{1}{\sqrt{e}}$, Theorem IV. will be more eligible for arithmetical calculation than Theorem II. It is obvious also, from the same article, that no more than one of those values need be computed in order to obtain the value of d. Putting, therefore, ζ for the value of z corresponding to $y = \frac{1}{\sqrt{e}}$, and $S = \begin{cases} \sqrt{e + ee} \\ + \frac{1}{2e} A' - \frac{1}{2.4e^2} B' + \frac{3}{2.4.6e^3} C', \text{ \&c.} \end{cases}$ (A', B', C', &c. being as there specified,) we have $\zeta = S - d$; and this value being written for ζ in the equation $d = 1 + e - 2\zeta$, it becomes $d = 1 + e + 2d - 2S$; from which we have $d = 2S - 1 - e$, which is another convenient *formula* for computing the value of d.

28. If the diagonal line of quantities in which l enters, either in the value of z or $z + d$, before referred to, were written by itself, and the remaining perpendicular columns summed in the manner by me described in the Philos. Trans. for 1798, p. 548 *et seq.* the value of d might be obtained in a pair of

series, each of them converging by the powers of $\frac{1}{ee}$. But the advantage of computing by such a pair of series, instead of the single one above described, is less than might at first be imagined; for, in order to have a result true to the same number of figures, about the same number of terms must be computed, whether of the single series or of the pair. Since, therefore, the advantage obtained by such a transformation lies not in the literal powers, it can have place only in the coefficients; and there it may be very considerable.

I now proceed to the investigation of a pair of series for computing the value of the constant quantity d, each of which converges by the powers of $\frac{1}{aa}$.

29. If the transverse and conjugate semi-axes of an hyperbola are denoted by a and 1, respectively, the ordinate by y, and the corresponding arch by z; and if the eccentricity, $\sqrt{aa+1}$, be denoted by e, and $1+eeyy$ be put $=uu$; then, as I have shewn in the Philos. Trans. for 1802, p. 457, will

$$z = \begin{cases} \sqrt{uu+aa} + \frac{1}{2}\mathrm{A} + \frac{3}{2.4}\mathrm{B} + \frac{3.5}{2.4.6}\mathrm{C} + \frac{3.5.7}{2.4.6.8}\mathrm{D}, \text{ \&c.} \\ -d; \end{cases}$$

$$\text{where } \mathrm{A} = \frac{1}{a}\,\mathrm{H.L.}\,\frac{\sqrt{(aa+uu)}-a}{u},$$

$$\mathrm{B} = \frac{-\sqrt{(uu+aa)}}{2aa\,uu} - \frac{\mathrm{A}}{2aa},$$

$$\mathrm{C} = \frac{-\sqrt{(uu+aa)}}{4aa\,u^4} - \frac{3\mathrm{B}}{4aa},$$

$$\mathrm{D} = \frac{-\sqrt{(uu+aa)}}{6aa\,u^6} - \frac{5\mathrm{C}}{6aa},$$

&c. &c.

and d, the constant quantity to be subtracted from the series, is (see p. 462 of the Philos. Trans. for 1802, and Art. 25;)

equal to the difference between the lengths of the infinite arch and its asymptote.

Now, when $y = o$, then $z = o$, and u becomes $= 1$; and the value of the series, in this case, is the value of d; and this value, if the terms of the series are ranged one under another according to the powers of $\frac{1}{aa}$, will stand as below, viz. $d =$

$$\begin{array}{llll}
\sqrt{1+aa} + \frac{A}{2} & & & \\
- \frac{3\sqrt{(1+aa)}}{2.4.2aa} & - \frac{3A}{2.4.2a^2} & & \\
- \frac{3.5\sqrt{(1+aa)}}{2.4.6.4aa} & + \frac{3.5.3\sqrt{(1+aa)}}{2.4.6.4.2a^4} & + \frac{3.5.3A}{2.4.6.4.2a^4} & \\
- \frac{3.5.7\sqrt{(1+aa)}}{2.4.6.8.6aa} & + \frac{3.5.7.5\sqrt{(1+aa)}}{2.4.6.8.6.4a^4} & - \frac{3.5.7.5.3\sqrt{(1+aa)}}{2.4.6.8.6.4.2a^6} & - \frac{3.5.7.5.3A}{2.4.6.8.6.4.2a^6} \\
\&c. & \&c. & \&c. & \&c.
\end{array}$$

Here, in the diagonal line of quantities which have the common factor A, we find the geometrical progression $\frac{1}{aa}, \frac{1}{a^4}, \frac{1}{a^6}$, &c. so that, when a is much greater than 1, this series will converge very swiftly. The same progression has place also in the perpendicular columns of quantities which remain after the diagonal line is taken away; in all which columns we find the constant factor $\sqrt{1+aa}$. If, therefore, the sums of the very slowly converging numeral series

$$\frac{3}{2.4.2} + \frac{3.5}{2.4.6.4} + \frac{3.5.7}{2.4.6.8.6} + \frac{3.5.7.9}{2.4.6.8.10.8}, \text{ \&c.}$$

$$\frac{3.5.3}{2.4.6.4.2} + \frac{3.5.7.5}{2.4.6.8.6.4} + \frac{3.5.7.9.7}{2.4.6.8.10.8.6} + \frac{3.5.7.9.11.9}{2.4.6.8.10.12.10.8}, \text{ \&c.}$$

$$\frac{3.5.7.5.3}{2.4.6.8.6.4.2} + \frac{3.5\,7.9.7.5}{2.4.6.8.10.8.6.4} + \frac{3.5.7.9.11.9.7}{2.4.6.8.10.12.10.8}, \text{ \&c.}$$

are taken (and they may easily be computed by the method explained in the Philos. Trans. for 1798, p. 547 to 550,) and

denoted by the Roman letters a, b, c,* &c. respectively, we shall have $d = \begin{cases} A \times : \frac{1}{2} - \frac{3}{2.4.2a^2} + \frac{3.5.3}{2.4.6.4.2a^4} - \frac{3.5.7.5.3}{2.4.6.8.6.4.2a^6}, \text{ \&c.} \\ + \sqrt{1+aa} \times : 1 - \frac{a}{2a} + \frac{b}{a^2} - \frac{c}{a^3}, \text{ \&c.} \end{cases}$

The law of continuation *ad infinitum* is very evident in the first of this pair of series; and although it is not so in the second, still it is obvious that b is less than a, c than b, &c. *ad infinitum*. And since the numeral coefficients of each of these series may be expressed in decimals, and their logarithms written out ready for use, an arithmetical calculation by this pair of series, when a is considerably greater than 1, will be much easier than by either of the single series given in Art. 25.

But this pair of series may be transformed into another pair converging by the same powers of a, yet of a simpler form, and therefore more convenient for arithmetical calcution. The operations † to be performed on this occasion are as follows.

30. The H. L. of $(\sqrt{1+aa} - a)$, which enters into the value of A, is $= -$ H. L. $(a + \sqrt{1+aa})$; and this logarithm, expressed in correct descending series, is $-$ H. L. $2a, -\frac{1}{2.2aa} + \frac{3}{2.4.4a^4} - \frac{3.5}{2.4.6.6a^6}$, &c. A, therefore, being $= \frac{1}{a}$ H. L. $(\sqrt{1+aa} - a)$, is $= -\frac{1}{a}$ H. L. $2a, -\frac{1}{2.2a^3} + \frac{3}{2.4.4a^5} - \frac{3.5}{2.4.6.6a^7}$, &c. $= -\frac{1}{a}(\lambda + l) - \frac{1}{2.2a^3} + \frac{3}{2.4.4a^5} - \frac{3.5}{2.4.6.6a^7}$, &c. λ being put for

* The values of these letters are $\frac{1}{2} + \frac{1}{4}$ H. L. 2, $\frac{1}{16} + \frac{7}{16}$ H. L. 2, and $\frac{17+51 \text{ H. L. } 2}{128}$, respectively. See Philos. Trans. for 1798, p. 538.

† See Philos. Trans. for 1798, p. 557 and 558; and for 1800, p. 87 and 88.

H. L. of z, and l for H. L. of a, for the sake of brevity, and of facility in the comparison of the result of the present operation with one which is next to be made. The multiplication of this series equivalent to A, by its proper factor, may stand thus:

$$-\frac{1}{a}(\lambda+l)-\frac{1}{2.2a^3}+\frac{3}{2.4.4a^5}-\frac{3.5}{2.4.6.6a^7},\ \&c.$$
$$\frac{1}{2}-\frac{3}{2.4.2a^2}+\frac{3.5.3}{2.4.6.4.2a^4}-\frac{3.5.7.5.3}{2.4.6.8.6.4.2a^6},\ \&c.$$

$$-\frac{1}{2a}(\lambda+l)-\frac{1}{2.4a^3}+\frac{3}{4.4.4a^5}-\frac{3.5}{4.4.6.6a^7},\ \&c.$$
$$+\frac{3(\lambda+l)}{2.4.2a^3}+\frac{3}{2.4.4.2a^5}-\frac{3.3}{2.4.4.4.4a^7},\ \&c.$$
$$-\frac{3.5.3(\lambda+l)}{2.4.6.4.2a^5}-\frac{3.5.3}{2.4.4.6.4.2a^7},\ \&c.$$
$$+\frac{3.5.7.5.3(\lambda+l)}{2.4.6.8.6.4.2a^7},\ \&c.$$

The value of $\sqrt{1+aa}$ also, in descending series, is $a+\frac{1}{2a}-\frac{1}{2.4a^3}+\frac{3}{2.4.6a^5}$, &c. and the multiplication of this series by its proper factor may be made as below:

$$a+\frac{1}{2a}-\frac{1}{2.4a^3}+\frac{3}{2.4.6a^5},\ \&c.$$
$$1-\frac{z}{aa}+\frac{b}{a^4}-\frac{c}{a^6},\ \&c.$$

$$a+\frac{1}{2a}-\frac{1}{2.4a^3}+\frac{3}{2.4.6a^5},\ \&c.$$
$$-\frac{z}{a}-\frac{z}{2a^3}+\frac{z}{2.4a^5},\ \&c.$$
$$+\frac{b}{a^3}+\frac{b}{2a^5},\ \&c.$$
$$-\frac{c}{a^5},\ \&c.$$

Now if the series $-\frac{l}{2a}+\frac{3l}{2.4a^3}-\frac{3.5.3l}{2.4.6.4.2a^5}$, &c. (which is

found in a diagonal line,) be taken from the first of these products, all the coefficients of the remaining terms (since $\lambda = \text{H. L. } 2$,) will be known quantities; and consequently all the remaining terms in each perpendicular column may be added together, and to their like quantities in the second product; so that the new pair of series expressing the value of d, will be this, viz.

$$\begin{cases} a - \frac{A}{a} + \frac{B}{a^3} - \frac{C}{a^5}, \text{ \&c.} \\ -\frac{l}{2a} + \frac{3l}{2.4.2a^3} - \frac{3.5.3l}{2.4.6.4.2a^5}, \text{ \&c.} \end{cases}$$

where the values of A, B, and C, are 0·44314718, 0·05680519, and 0·02183137, respectively. The law which the coefficients of the logarithmic series observe is evident; the law, which the coefficients (A, B, C, &c.) of the other series observe, will be discovered by the following process.

31. It appears by Mac Laurin's *Fluxions*, Art. 808, and by the Philos. Trans. for 1809, p. 462, that $d = 1{\cdot}57$, &c. $\times : \frac{aa}{2} - \frac{3a^4}{2.2.4} + \frac{3.3.5a^6}{2.2.4.4.6} - \frac{3.3.5.5.7a^8}{2.2.4.4.6.6.8}$, &c.; where it is evident that the value of d depends entirely upon that of a, and that these two quantities must be constant or vary together. Therefore, supposing these quantities to vary, and taking the fluxion on both sides of the equation, we have

$$\dot{d} = 1{\cdot}57 \text{ \&c. } \dot{a} \times : a - \frac{3a^3}{2.2} + \frac{3.3.5a^5}{2.2.4.4} - \frac{3.3.5.5.7a^7}{2.2.4.4.6.6}, \text{ \&c.}$$

This equation divided by a, and the fluxion taken again on both sides, making $\dot{a}$ constant, gives

$$\frac{\ddot{d}}{a} - \frac{\dot{a}\dot{d}}{a\dot{a}} = 1{\cdot}57 \text{ \&c. } \dot{a}\dot{a} * : \times - \frac{3a}{2} + \frac{3.3.5a^3}{2.4.4} - \frac{3.3.5.5.7a^5}{2.2.4.4.6}, \text{ \&c.}$$

Here we find the denominators of the fractional coefficients of the terms on the second side of the equation to be the very

same as those in the original equation; so that, if we can find herefrom another equation in which the numerators also shall be the same as those which were there found, we shall obtain an important point. Now this may easily be done as follows: Multiply the last equation by a, and take the fluents, and there will be

$$\dot{d} - f\frac{a\ddot{d}}{a} = 1{\cdot}57 \text{ \&c. } \dot{a} \times : -\frac{a^3}{2} + \frac{3{\cdot}3a^5}{2{\cdot}2{\cdot}4} - \frac{3{\cdot}3{\cdot}5{\cdot}5a^7}{2{\cdot}2{\cdot}4{\cdot}4{\cdot}6}, \text{ \&c.}$$

Divide this equation by a^3, and take the fluents again, and we shall have

$$f\frac{\dot{d}}{a^3} - f\frac{1}{a^3}f\frac{a\ddot{d}}{a} = 1{\cdot}57 \text{ \&c. } \times : -\frac{a}{2} + \frac{3a^3}{2{\cdot}2{\cdot}4} - \frac{3{\cdot}3{\cdot}5a^5}{2{\cdot}2{\cdot}4{\cdot}4{\cdot}6}, \text{ \&c.}$$

which is evidently $= -\frac{d}{a}$.

This new equation will be fitted for our purpose by taking the fluxion on both sides, multiplying by a^3, and then taking the fluxions again; which operations being performed, (remembering that $\dot{a}$ must still be made constant,) and the terms brought all to one side, and properly arranged, the result will be $\dot{a}\dot{a}d + (\frac{1}{a} - a)\,\dot{a}\dot{d} - (aa + 1)\,\ddot{d} = 0$.

82. The values of d, $\dot{d}$, and $\ddot{d}$, are now to be taken in terms of the pair of series obtained in Art. 80, and substituted in the equation last found. And, although the coefficients of the logarithmic series, and the law which they observe *ad infinitum*, be already discovered, yet, for the sake of brevity, I denote the first, second, third, &c. of them by α, ζ, γ, &c. respectively. In this notation we have

$$d = \begin{cases} a - \frac{A}{a} + \frac{B}{a^3} - \frac{C}{a^5} + \frac{D}{a^7} - \frac{E}{a^9}, \text{ \&c.} \\ + l \times : \frac{-\alpha}{a} + \frac{\zeta}{a^3} - \frac{\gamma}{a^5} + \frac{\delta}{a^7} - \frac{\iota}{a^9}, \text{ \&c.} \end{cases}$$

$$\dot{d} = \begin{cases} \dot{a} \times : 1 + \frac{A}{aa} - \frac{3B}{a^3} + \frac{5C}{a^4} - \frac{7D}{a^5} + \frac{9E}{a^{10}}, \text{ &c.} \\ + \dot{a} \times : = \frac{a}{aa} + \frac{c}{a^3} - \frac{7}{a^4} + \frac{9}{a^5} - \frac{11}{a^{10}}, \text{ &c.} \\ + \dot{a}\, l \times : \frac{a}{aa} - \frac{3c}{a^3} + \frac{5\gamma}{a^4} - \frac{7\delta}{a^5} + \frac{9\iota}{a^{10}}, \text{ &c.} \end{cases}$$

$$\ddot{d} = \dot{a}\dot{a} \begin{cases} - \frac{2A}{a^3} + \frac{3 \cdot 4B}{a^4} - \frac{5 \cdot 6C}{a^5} + \frac{7 \cdot 8D}{a^6} - \frac{9 \cdot 10E}{a^{11}}, \text{ &c.} \\ + \frac{2a}{a^3} - \frac{4c}{a^4} + \frac{6\gamma}{a^5} - \frac{8\delta}{a^6} + \frac{10\iota}{a^{11}}, \text{ &c.} \\ + \frac{a}{a^3} - \frac{3c}{a^4} + \frac{5\gamma}{a^5} - \frac{7\delta}{a^6} + \frac{9\iota}{a^{11}}, \text{ &c.} \\ + l : \times \frac{-2a}{a^3} + \frac{3 \cdot 4c}{a^4} - \frac{5 \cdot 6\gamma}{a^5} + \frac{7 \cdot 8\delta}{a^6} - \frac{9 \cdot 10\iota}{a^{11}}, \text{ &c.} \end{cases}$$

The last two equations, more concisely expressed, are

$$\dot{d} = \dot{a} \begin{cases} 1 + \frac{A-a}{aa} - \frac{3B-c}{a^4} + \frac{5C-\gamma}{a^5} - \frac{7D-\delta}{a^6} + \frac{9E-\iota}{a^{10}}, \text{ &c.} \\ + l \times : \frac{a}{aa} - \frac{3c}{a^4} + \frac{5\gamma}{a^5} - \frac{7\delta}{a^6} + \frac{9\iota}{a^{10}}, \text{ &c.} \end{cases}$$

$$\ddot{d} = \dot{a}\dot{a} \begin{cases} - \frac{2A-3a}{a^3} + \frac{3 \cdot 4B-7c}{a^4} - \frac{5 \cdot 6C-11\gamma}{a^5} + \frac{7 \cdot 8D-15\delta}{a^6} - \frac{9 \cdot 10E-19\iota}{a^{11}}, \text{ &c.} \\ + l \times : \frac{-2a}{a^3} + \frac{3 \cdot 4c}{a^4} - \frac{5 \cdot 6\gamma}{a^5} + \frac{7 \cdot 8\delta}{a^6} - \frac{9 \cdot 10\iota}{a^{11}}, \text{ &c.} \end{cases}$$

These values of d, $\dot{d}$, and $\ddot{d}$, being written for them in the equation $\dot{a}\dot{a}d + (\frac{1}{a} - a)\,\dot{a}\dot{d} - (aa + 1)\,\ddot{d} = o$, and the algebraic and logarithmic terms severally collected together in two parcels, and the whole divided by $\dot{a}\dot{a}$, we have $o =$

$$a - \frac{A}{a} + \frac{B}{a^3} - \frac{C}{a^5} + \frac{D}{a^7} - \frac{E}{a^9}, \text{ &c.}$$

$$- a - \frac{A-a}{a} + \frac{3B-c}{a^3} - \frac{5C-\gamma}{a^5} + \frac{7D-\delta}{a^7} - \frac{9E-\iota}{a^9}, \text{ &c.}$$

$$+ \frac{1}{a} + \frac{A-a}{a^3} - \frac{3B-c}{a^5} + \frac{5C-\gamma}{a^7} - \frac{7D-\delta}{a^9}, \text{ &c.}$$

$$+ \frac{2A-3a}{a} - \frac{3 \cdot 4B-7c}{a^3} + \frac{5 \cdot 6C-11\gamma}{a^5} - \frac{7 \cdot 8D-15\delta}{a^7} + \frac{9 \cdot 10E-19\iota}{a^9}, \text{ &c.}$$

$$+ \frac{2A-3a}{a^3} - \frac{3 \cdot 4B-7c}{a^5} + \frac{5 \cdot 6C-11\gamma}{a^7} - \frac{7 \cdot 8D-15\delta}{a^9}, \text{ &c.}$$

$$+l\left\{\begin{array}{l} -\frac{\alpha}{a}+\frac{\zeta}{a^3}-\frac{\gamma}{a^5}+\frac{\delta}{a^7}-\frac{\iota}{a^9}, \text{ \&c.} \\ -\frac{\alpha}{a}+\frac{3\zeta}{a^3}-\frac{5\gamma}{a^5}+\frac{7\delta}{a^7}-\frac{9\iota}{a^9}, \text{ \&c.} \\ \quad +\frac{\alpha}{a^3}-\frac{3\zeta}{a^5}+\frac{5\gamma}{a^7}-\frac{7\delta}{a^9}, \text{ \&c.} \\ +\frac{2\alpha}{a}-\frac{3.4\zeta}{a^3}+\frac{5.6\gamma}{a^5}-\frac{7.8\delta}{a^7}+\frac{9.10\iota}{a^9}, \text{ \&c.} \\ \quad +\frac{2\alpha}{a^3}-\frac{3.4\zeta}{a^5}+\frac{5.6\gamma}{a^7}-\frac{7.8\delta}{a^9}, \text{ \&c.} \end{array}\right.$$

Now, unless each of these parcels of quantities, as well as their sum, be universally $= v$, this equation will be of no use to us. And if it can be proved, that the coefficients of the terms in the one series are, each of them, $= o$, then it will follow, that each of the coefficients of the terms in the other series is also $= o$. But each of the coefficients of the terms in the logarithmic series is $= o$; which may be proved thus: when the like quantities in this series are added together, the first term will vanish; and the coefficients of the second, third, fourth, fifth, &c. terms (without the common factor l,) will be $3\alpha - 2.4\zeta$, $-3.5\zeta + 4.6\gamma$, $5.7\gamma - 6.8\delta$, $-7.9\delta + 8.10\iota$, &c. respectively; and the law of continuation is obvious. But by the law (see Art. 29 and 30;) which the coefficients α, ζ, γ, δ, &c. are known to observe, $\frac{3\alpha}{2.4}$ is $= \zeta$, $\frac{3.5\zeta}{4.6} = \gamma$, $\frac{5.7\gamma}{6.8} = \delta$, $\frac{7.9\delta}{8.10} = \iota$, &c.; and therefore, $3\alpha - 2.4\zeta = o$, $3.5\zeta - 4.6\gamma = o$, $5.7\gamma - 6.8\delta = o$, $7.9\delta - 8.10\iota = o$, &c.

The value of α, viz. $\frac{1}{2}$, which was discovered in Art. 29, is found also in the algebraic series, as will presently appear. For, adding like quantities together, the first term of this series also will vanish; and the coefficients of the second, third, fourth, fifth, sixth, &c. will be as follows:

Coeff. of 2d term $1-2\alpha$,

3d $-(3.3-1)\,B+1.3A-4\alpha+6\epsilon$,

4th $+(5.5-1)\,C-3.5B+8\epsilon-10\gamma$,

5th $-(7.7-1)\,D+5.7C-12\gamma+14\delta$,

6th $+(9.9-1)\,E-7.9D+16\delta-18\epsilon$,

&c. &c.

And, putting each of these coefficients $=0$, (since the whole series is now known to be $=0$,) writing 2.4 for its equal $3.3-1$, 4.6 for $5.5-1$, 6.8 for $7.7-1$, &c. we obtain from these equations

$$\alpha=\tfrac{1}{2},$$

$$B=\tfrac{1.3}{2.4}A-\tfrac{\alpha}{2}+\tfrac{3\epsilon}{2.2},$$

$$C=\tfrac{3.5}{4.6}B-\tfrac{\epsilon}{3}+\tfrac{5\gamma}{3.4},$$

$$D=\tfrac{5.7}{6.8}C-\tfrac{\gamma}{4}+\tfrac{7\delta}{4.6},$$

$$E=\tfrac{7.9}{8.10}D-\tfrac{\delta}{5}+\tfrac{9\epsilon}{5.8},$$

&c. &c.

the law of continuation being very evident.

The value of A, which is not discovered by this process, was found in Art. 30, and is $=\frac{1}{4}+$ H. L. s. And the decimal values of these coefficients are as below, viz.

$$A=0.44814718,$$

$$B=0.05680519,$$

$$C=0.02183137,$$

$$D=0.01154459,$$

$$E=0.00714200,$$

&c. &c.

33. Thus, by the common application of Sir Isaac Newton's doctrine of fluxions and infinite series, without any assistance

from, or regard to, LANDEN's theorem, we have obtained a pair of series for computing the value of d, which converge by the powers of $\frac{1}{aa}$, and of which we can readily find as many terms as we please. And, by a similar process, (as was observed in Art. 26,) may EULER's series for computing the quadrantal arch of an ellipsis be obtained, without any use of FAGNANI's theorem, or the "*tentative methods*," and "*strange artifices*," as Mr. WOODHOUSE* calls them, which appear in EULER's paper.

34. If we look back to Art. 18 and 20 of this Paper, we shall find that, when the transverse and conjugate semi-axes of an hyperbola are denoted by 1 and $\frac{1}{a}$, respectively, (which hyperbola will be similar to that of which the semi-axes are a and 1,) the convergency of the first series, derived from LANDEN's theorem, will be by the powers of the fraction $\left(\frac{\frac{1}{a}}{\frac{1}{a}+\sqrt{(1+\frac{1}{aa})}}=\right)\frac{1}{1+\sqrt{(aa+1)}}$, assisted by coefficients. And the same rate of convergency will obtain in the series given in Art. 29, by putting $y=\frac{1}{\sqrt{e}}$; for then au, by which the terms of that series are divided, will be $=1+e$, e being $=\sqrt{aa+1}$: so that, when aa is less than $1+\sqrt{aa+1}$, the terms of the single series will decrease swifter than the terms of the pair of series; and consequently half as many terms of the former as the latter will give a result equally near the truth. The two quantities aa and $1+\sqrt{aa+1}$ are equal when $aa=3$; and hence it appears, that the proper use of the pair of series above found, is, when a is considerably

* See Philos. Trans. for 1804, p. 235.

greater than $\sqrt{3}$. When a is a large number, about as many terms of the single series as of the pair must be computed to have the results true to the same number of figures; yet the operation by the pair will be by much the easiest.

35. If both sides of the equation

$$d = \begin{cases} a - \frac{A}{a} + \frac{B}{a^3} - \frac{C}{a^5} + \frac{D}{a^7}, \text{ \&c.} \\ -l \times : \frac{a}{a} - \frac{c}{a^3} + \frac{\gamma}{a^5} - \frac{\delta}{a^7}, \text{ \&c.} \end{cases}$$

were divided by a, and if $\frac{d}{a}$ were put $= {}'d$, and $\frac{1}{a} = b$; then, (since H. L. of b would be $= -l$,) we should have

$${}'d = \begin{cases} 1 - Abb + Bb^4 - Cb^6 + Db^8, \text{ \&c.} \\ -l \times : abb - \zeta b^4 + \gamma b^6 - \delta b^8, \text{ \&c.} \end{cases}$$

where ${}'d$ denotes the difference between the lengths of the infinite arch and the asymptote, (and the difference also between the values of the ascending and descending series for computing the arch,) of an hyperbola of which the semi-axes are 1 and b, respectively.

36. It was observed in Art. 12, that the fluxion there given, of an hyperbolic arch, is as capable of transformation as that which has been commonly used for the rectification of an ellipsis: so also are those which I have used in the Philos. Trans. for 1809, p. 451 and 455, and from which series converging by the powers of $\left(\frac{\sigma - a}{\sigma + a}\right)^2$, &c. may easily be obtained; but to treat of such transformations is not only foreign from my present design, but would extend this paper to a considerable length. I shall therefore only point out, by a few examples, the great advantage, in many cases, of computing by descending series, and then conclude.

37. *Example* 1. The transverse and conjugate semi-axes

of an hyperbola are 7 and 1, respectively; it is required to find the lengths of three arches of which the three ordinates are 10, 20, and 30.

Setting aside the circuitous method of rectifying the hyperbola by means of two ellipses, if one was to think of computing these arches by means of a similar hyperbola, and by the theorem (δ) given in Art. 9, (which is a very useful *Formula* within the limits specified in Art. 13,) he would quickly perceive, that the ascending series, by which the value of G is obtained, would converge very slowly; and therefore would make choice of some other method. Theorem IV. in my former tract (see Philos. Trans. for 1802, p. 454,) is a very convenient form to be used on this occasion, and is as follows:

Retaining the notation used in the beginning of Art. 29,

$$x = \begin{cases} e\sqrt{yy+1} \\ + \frac{1}{2e}A - \frac{1}{2.4e^3}B + \frac{3}{2.4.6e^5}C - \frac{3.5}{2.4.6.8e^7}D, \text{ \&c.} \\ - d; \end{cases}$$

where $A = \text{H.L.}\frac{\sqrt{(yy+1)}-1}{y}$,

$$B = \frac{-\sqrt{(yy+1)}}{2yy} - \frac{A}{2},$$

$$C = \frac{-\sqrt{(yy+1)}}{4y^4} - \frac{3B}{4},$$

$$D = \frac{-\sqrt{(yy+1)}}{6y^6} - \frac{5C}{6},$$

&c. &c.

The first part of the work may be to find the value of d, which may easily be done by the pair of series given in Art. 32; but, since a computation of it was made by two series, in p. 466, 467, and 468 of the volume here referred to, the ascending series being = 0·6960768, the descending series

$= 7{\cdot}4849912$, and their difference, d, $= 6{\cdot}7989144$; we need only to verify that work, which may easily be done by the *Formulæ* given in Art. 25 and 27, thus: denoting the value of the ascending series corresponding to the ordinate $\frac{1}{\sqrt{e}}$ by ζ, and the value of the descending series corresponding to the same ordinate by S, we have,

by Art. 25, $d = 1 + e - 2\zeta$, $\left\{\begin{array}{l} = 8{\cdot}0710678 \\ - 1{\cdot}2721536 \end{array}\right.$

$= 6{\cdot}7989142$;

by Art. 27, $d = 2S - 1 - e$, $\left\{\begin{array}{l} = 14{\cdot}8699824 \\ - 8{\cdot}0710678 \end{array}\right.$

$= 6{\cdot}7989146$;

the difference in the last figures of the results arising from the inaccuracy of decimal fractions, and being wholly inconsiderable.

The rest of the work may stand as follows: when y is $=$ 10, then, (a being $= 7$, and $= \sqrt{aa+1} = \sqrt{50}$,)

$$A = \text{H. L.}\frac{\sqrt{(yy+1)}-1}{y} = -0{\cdot}0998341,$$

$$B = \frac{-\sqrt{(yy+1)}}{4yy} - \frac{A}{2} = -0{\cdot}0003383,$$

$$C = \frac{-\sqrt{(yy+1)}}{4y^4} - \frac{1B}{4} = -0{\cdot}0000020;$$

of which terms two only are wanted to obtain a result true to seven places of figures. Hence we have

	+	−
$e\sqrt{yy+1}$ =	71·0633520,	$+\frac{1}{2e}A$ = 0·0070598,
$-\frac{1}{2.4e^3}B$ =	0·0000001,	$-d$ = 6·7989144,
sum of the posit. terms	71·0633521;	the sum — 6·8059737;
neg. term, & correc. —	6·8059737;	

the difference is 64·2573784 = the length of the first arch.

When $y = 20$, we have

$$A = \text{H. L.}\frac{\sqrt{(yy+1)}-1}{y} = -0{\cdot}0499794,$$

$$B = \frac{\sqrt{(yy+1)}}{2yy} - \frac{A}{2} = -0{\cdot}0000415;$$

of which two terms the first only is sufficient to give the result true to eight places of figures. We now have

	+	−
$e\sqrt{yy+1}$ =	141·5980226,	$+\frac{1}{2e}A$ = 0·0035341,
$-\frac{1}{2.4e^3}B$ =	0·0000000,	$-d$ = 6·7989144,
sum of the posit. terms	141·5980226;	the sum — 6·8024485;
neg. term & correc. —	6·8024485;	

the difference is 134·7955741 = the length of the second arch.

When y is = 30, the value of $A = \text{H. L.}\frac{\sqrt{(yy+1)}-1}{y}$ is — 0·0333274; and since the value of B (as appeared in the preceding operation,) need not be computed, we have

	+	−
$e\sqrt{yy+1}$ =	212·2498528,	$\frac{1}{2e}A$ = 0·0023566,
neg. term and correc. —	6·8019710,	$-d$ = 6·7989144;
length of the third arch	205·4485818.	sum 6·8012710;

It is now obvious that, to obtain the length of any greater arch of this hyperbola, the values of the algebraic quantity $e\sqrt{yy+1}$, and $\frac{1}{3e}$ A, the first term of the series, are all that need be computed; for the value of d, once found, serves for all. And if seven more arches of this hyperbola, corresponding to the ordinates 40, 50, 60, &c. to 100, were to be computed, this theorem would afford a striking instance of the great utility of descending series.

38. A *second example* might be, to find the lengths of ten arches of an equilateral hyperbola, of which the semi-axis is 1, when the ordinates are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

These arches may be computed by the new theorem given in Art. 9, in which the value of G is always given in ascending series; but that series, when y is much greater than 1, will converge very little faster than the powers of $\frac{1}{2}$: whereas, by using the theorem given in the Philos. Trans. for 1802, p. 458, viz.

$$z = \begin{cases} u - \frac{1}{2.3u^3} - \frac{3}{2.4.7u^5} - \frac{3.5}{2.4.6.11u^{11}} - \frac{3.5.7}{2.4.6.8.15u^{15}}, \text{ &c.} \\ -d, \end{cases}$$

(where z denotes the arch of an equilateral hyperbola of which the semi-axis is 1, u is $= \sqrt{1+2yy}$, and d is $= 0.5990701 9$;) the geometrical progressions which will have place in the series, for the respective ordinates, will be the powers of these fractions, viz. $\frac{1}{9}$, $\frac{1}{81}$, $\frac{1}{361}$, $\frac{1}{1089}$, $\frac{1}{2601}$, $\frac{1}{5329}$, $\frac{1}{9801}$, $\frac{1}{16641}$, $\frac{1}{26569}$, and $\frac{1}{40401}$.

39. In these examples the use and advantage of descending series appear: more examples of their utility might be given; and it might easily be shown, that there are cases in which

such series have the advantage, even when the ascending series have a good rate of convergency. I trust, however, that enough has been done in this Paper to satisfy all candid and competent judges of the matter, that *the Rectification of the Hyperbola by means of two Ellipses is more curious than useful;* that the advantage of computing by descending series, is, in many cases, very great; and that such series will often answer the end of a transformation without the trouble of making it.

VIII. *On a Combination of Oxymuriatic Gas and Oxygene Gas.* By Humphry Davy, *Esq. LL. D. Sec. R. S. Prof. Chem. R. I.*

Read February 21, 1811.

I SHALL beg permission to lay before the Society the account of some experiments on a compound of oxymuriatic gas and oxygene gas, which, I trust, will be found to illustrate an interesting branch of chemical enquiry, and which offer some extraordinary and novel results.

I was led to make these experiments in consequence of the difference between the properties of oxymuriatic gas prepared in different modes; it would occupy a great length of time to state the whole progress of this investigation. It will, I conceive, be more interesting that I should immediately refer to the facts; most of which have been witnessed by Members of this Body, belonging to the Committee of Chemistry of the Royal Institution.

The oxymuriatic gas prepared from manganese, either by mixing it with a muriate and acting upon it by sulphuric acid, or by mixing it with muriatic acid, is when the oxide of manganese is pure, and, whether collected over water or mercury, uniform in its properties; its colour is a pale yellowish green; water takes up about twice its volume; and scarcely gains any colour; the metals burn in it readily; it combines with hydrogene without any deposition of moisture: it does not

act on nitrous gas or muriatic acid, or carbonic oxide, or sulphureous gasses, when they have been carefully dried. It is the substance which I employed in all the experiments on the combinations of oxymuriatic gas, described in my last two papers.

The gas produced by the action of muriatic acid on the salts which have been called hyperoxymuriates, on the contrary, differs very much in its properties, according as the manner in which it is prepared and collected is different.

When much acid is employed to a small quantity of salt, and the gas is collected over water, the water becomes tinged of a lemon colour; but the gas collected is the same as that procured from manganese.

When the gas is collected over mercury, and is procured from a weak acid, and from a great excess of salt, by a low heat, its colour is a dense tint of brilliant yellow green, and it possesses properties entirely different from the gas collected over water.

It sometimes explodes during the time of its transfer from one vessel to another, producing heat and light, with an expansion of volume; and it may be always made to explode by a very gentle heat, often by that of the hand.*

* My brother, Mr. J. DAVY, from whom I receive constant and able assistance in all my chemical enquiries, had several times observed explosions, in transferring the gas from hyperoxymuriate of potash, over mercury, and he was inclined to attribute the phænomenon to the combustion of a thin film of mercury, in contact with a globule of gas. I several times endeavoured to produce the effect, but without success, till an acid was employed for the preparation of the gas, so diluted as not to afford it without the assistance of heat. The change of colour and expansion of volume, when the effect took place, immediately convinced me, that it was owing to a decomposition of the gas.

It is a compound of oxymuriatic gas and oxygene, mixed with some oxymuriatic gas. This is proved by the results of its spontaneous explosion. It gives off, in this process, from $\frac{1}{5}$ to $\frac{2}{5}$ its volume of oxygene, loses its vivid colour, and becomes common oxymuriatic gas.

I attempted to obtain the explosive gas in a pure form, by applying heat to a solution of it in water; but in this case, there was a partial decomposition; and some oxygene was disengaged, and some oxymuriatic gas formed. Finding that in the cases when it was most pure, it scarcely acted upon mercury, I attempted to separate the oxymuriatic gas with which it is mixed, by agitation in a tube with this metal; corrosive sublimate formed, and an elastic fluid was obtained, which was almost entirely absorbed by $\frac{1}{4}$ of its volume of water.

This gas in its pure form is so easily decomposable, that it is dangerous to operate upon considerable quantities.

In one set of experiments upon it, a jar of strong glass, containing 40 cubical inches, exploded in my hands with a loud report, producing light; the vessel was broken, and fragments of it were thrown to a considerable distance.

I analysed a portion of this gas, by causing it to explode over mercury in a curved glass tube, by the heat of a spirit lamp.

The oxymuriatic gas formed, was absorbed by water; the oxygene was found to be pure, by the test of nitrous gas.

50 parts of the detonating gas, by decomposition, expanded so as to become 60 parts. The oxygene, remaining after the absorption of the oxymuriatic gas, was about 20 parts. Several other experiments were made, with similar results. So that it may be inferred, that it consists of 2 in volume of oxymuriatic gas, and 1 in volume of oxygene; and the oxy-

gene in the gas is condensed to half its volume. Circumstances conformable to the laws of combination of gaseous fluids, so ably illustrated by M. Gay Lussac, and to the theory of definite proportions.

I have stated on a former occasion, that approximations to the numbers representing the proportions in which oxygene and oxymuriatic gas combine, are found in 7.5 and 30.9. And this compound gas contains nearly these quantities.*

The smell of the pure explosive gas somewhat resembles that of burnt sugar, mixed with the peculiar smell of oxymuriatic gas. Water appeared to take up eight or ten times its volume; but the experiment was made over mercury, which might occasion an error, though it did not seem to act on the fluid. The water became of a tint approaching to orange.

When the explosive gas was detonated with hydrogene, equal to twice its volume, there was a great absorption, to more than $\frac{1}{3}$, and solution of muriatic acid was formed; when the explosive gas was in excess, oxygene was always expelled, a fact demonstrating the stronger attraction of hydrogene for oxymuriatic gas than for oxygene.

I have said that mercury has no action upon this gas in its purest form at common temperatures. Copper and anti-

* In page 245 of the Phil. Trans. for 1810, I have mentioned that the specific gravity of oxymuriatic gas, is between 74 and 75 grains per 100 cubical inches. The gas that I weighed, was collected over water and procured from hyperoxymuriate of potash, and at that time I conceived, that this elastic fluid did not differ from the oxymuriatic gas from manganese, except in being purer. It probably contained some of the new gas; for I find that the specific gravity of pure oxymuriatic gas from manganese, and muriatic acid is to that of common air, as 244 to 100 Taking this estimation, the specific gravity of the new gas will be about 238, and the number representing the proportion in which oxymuriatic gas combines, from this estimation, will be rather higher than is stated above.

mony, which so readily burn in oxymuriatic gas, did not act upon the explosive gas in the cold: and when they were introduced into it, being heated, it was instantly decomposed, and its oxygene set free; and the metals burnt in the oxymuriatic gas.

When sulphur was introduced into it, there was at first no action, but an explosion soon took place: and the peculiar smell of oxymuriate of sulphur was perceived.

Phosphorus produced a brilliant explosion, by contact with it in the cold, and there was produced phosphoric acid and solid oxymuriate of phosphorus.

Arsenic introduced into it did not inflame; the gas was made to explode, when the metal burnt with great brilliancy in the oxymuriatic gas.

Iron wire introduced into it did not burn, till it was heated so as to produce an explosion, when it burnt with a most brilliant light in the decomposed gas.

Charcoal introduced in it ignited, produced a brilliant flash of light, and burnt with a dull red light, doubtless owing to its action upon the oxygene mixed with the oxymuriatic gas.

It produced dense red fumes when mixed with nitrous gas, and there was an absorption of volume.

When it was mixed with muriatic acid gas, there was a gradual diminution of volume. By the application of heat the absorption was rapid, oxymuriatic gas was formed, and a dew appeared on the sides of the vessel.

These experiments enable us to explain the contradictory accounts that have been given by different authors of the properties of oxymuriatic gas.

That the explosive compound has not been collected before,

is owing to the circumstance of water having been used for receiving the products from hyperoxymuriate of potash, and unless the water is highly saturated with the explosive gas, nothing but oxymuriatic gas is obtained; or to the circumstance of too dense an acid having been employed.

This substance produces the phænomena which Mr. CHENEVIX, in his able paper on oxymuriatic acid, referred to the hyperoxygenised muriatic acid; and they prove the truth of his ideas respecting the possible existence of a compound of oxymuriatic gas, and oxygene in a separate state.

The explosions produced in attempts to procure the products of hyperoxymuriate of potash by acids are evidently owing to the decomposition of this new and extraordinary substance.

All the conclusions which I have ventured to make respecting the undecompounded nature of oxymuriatic gas, are, I conceive, entirely confirmed by these new facts.

If oxymuriatic gas contained oxygene, it is not easy to conceive, why oxygene should be afforded by this new compound to muriatic gas, which must already contain oxygene in intimate union. Though on the idea of muriatic acid being a compound of hydrogene and oxymuriatic gas, the phænomena are such as might be expected.

If the power of bodies to burn in oxymuriatic gas depended upon the presence of oxygene, they all ought to burn with much more energy in the new compound; but copper and antimony, and mercury, and arsenic, and iron, and sulphur have no action upon it, till it is decomposed; and they act then according to their relative attractions on the oxygene, or on the oxymuriatic gas.

There is a simple experiment which illustrates this idea;

Let a glass vessel containing brass foil be exhausted, and the new gas admitted, no action will take place; throw in a little nitrous gas, a rapid decomposition occurs, and the metal burns with great brilliancy.

Supposing oxygene and oxymuriatic gas to belong to the same class of bodies; the attraction between them might be conceived very weak, as it is found to be, and they are easily separated from each other, and made repulsive by a very low degree of heat.

The most vivid effects of combustion known, are those produced by the condensation of oxygene or oxymuriatic gas; but in this instance, a violent explosion with heat and light are produced by their separation, and expansion, a perfectly novel circumstance in chemical philosophy.

This compound destroys dry vegetable colours, but first gives them a tint of red. This and its considerable absorbability by water would incline one to adopt Mr. CHENEVIX's idea that it approaches to an acid in its nature. It is probably combined with the peroxide of potassium in the hyperoxymuriate.

That oxymuriatic gas and oxygene combine and separate from each other with such peculiar phænomena, appears strongly in favour of the idea of their being distinct, though analogous species of matter. It is certainly possible to defend the hypothesis that oxymuriatic gas consists of oxygene united to an unknown basis; but it would be possible likewise to defend the speculation that it contains hydrogene.

Like oxygene it has not yet been decomposed; and I sometime ago made an experiment, which, like most of the others I have brought forward, is very adverse to the idea of its containing oxygene.

I passed the solid oxymuriate of phosphorus in vapour, and oxygene gas together through a green glass tube heated to redness.

A decomposition took place, and phosphoric acid was formed, and oxymuriatic gas was expelled.

Now, if oxygene existed in the oxymuriate of phosphorus, there is no reason why this change should take place. On the idea of oxymuriatic gas being undecompounded, it is easily explained. Oxygene is known to have a stronger attraction for phosphorus than oxymuriatic gas has, and consequently ought to expel it from this combination.

As the new compound in its purest form is possessed of a bright yellow green colour, it may be expedient to designate it by a name expressive of this circumstance, and its relation to oxymuriatic gas. As I have named that elastic fluid Chlorine, so I venture to propose for this substance the name Euchlorine, or Euchloric gas from εὐ and χλωρος. The point of Nomenclature I am not, however, inclined to dwell upon. I shall be content to adopt any name that may be considered as most appropriate by the able chemical philosophers attached to this Society.

*** In page 27, line 11, of the Bakerian lecture, for "water separated and LIBAVIUS's liquor was formed," read "a compound of water and LIBAVIUS's liquor separated." In page 21, it is stated that magnesia is not decomposed by oxymuriatic gas at a red heat. From some experiments of M. M. GAY LUSSAC, and THENARD, Bullet. de la Societ. Phil. Mai, 1810, it appears that oxygene is procured by passing oxymuriatic gas over magnesia, at a high temperature, and that a muriate indecomposable by heat is proved. They attribute the presence of this oxygene to the decomposition of the acid, but according to all analogies, it must arise from the decomposition of the earth.

VIII. *Experiments to prove that Fluids pass directly from the Stomach to the Circulation of the Blood, and from thence into the Cells of the Spleen, the Gall Bladder, and Urinary Bladder, without going through the Thoracic Duct.* By Everard Home, *Esq. F. R. S.*

Read January 31, 1811.

Having on a former occasion laid before the Society some experiments, to prove that fluids pass directly from the cardiac portion of the stomach, so as to arrive at the circulation of blood without going through the thoracic duct, the only known channel by which liquids can arrive there; the present experiments are brought to confirm that opinion: but in stating them, I wish to correct an error I was led into, in believing that the spleen was the channel by which they are conveyed.

At the time I made my former communications, I was conscious that the facts I had ascertained were only sufficient to open a new field of enquiry; but as I might never be able to make a further progress in an investigation, beset with so many difficulties, I thought it right to put them on record. Since that time I have lost no opportunity of devising new experiments to elucidate this subject; and the circumstance of Mr. Brodie, the assistant of my philosophical as well as professional labours, having tied the thoracic duct in some experiments which will come before the Society, suggested to me the idea, that if the thoracic duct was tied, and proper experi-

ments made, there could be no difficulty in ascertaining whether there was any other channel between the stomach and the circulation of the blood.

With this view I instituted the following experiment, which was made on the 29th of September 1810, by Mr. BRODIE, assisted by Mr. WILLIAM BRANDE and Mr. GATCOMBE. I was unavoidably prevented from being present during the time of the experiment.

Experiment 1.

A ligature was passed round the thoracic duct of a rabbit, just before it enters at the junction between the left jugular and subclavian veins: an ounce of strong infusion of rhubarb was then injected into the stomach. In three quarters of an hour some urine was voided, in which rhubarb was distinctly detected, by the addition of potash. An hour and a quarter after the injection of the rhubarb the animal was killed: a dram and half of urine was found in the bladder highly tinged with rhubarb, and the usual alteration of colour took place on the addition of potash. The coats of the thoracic duct had given way opposite the middle dorsal vertebra, and nearly an ounce of chyle was found effused into the cavity of the thorax, beside a considerable quantity in the cellular membrane of the posterior mediastinum. Above the ruptured part the thoracic duct was entire, much distended with chyle; and on tracing it upwards, the termination of the duct in the vein was found to be completely secured by the ligature. The lacteal and lymphatic vessels had given way in several parts of the abdomen, and chyle and lymph were extravasated underneath the peritoneum.

In this and the following experiments the infusion of rhubarb was employed in preference to the prussiate of potash, in consequence of its having been found in those I formerly made, that one drop of tincture of rhubarb could be detected in half an ounce of serum, and nothing less than a quarter of a grain of prussiate of potash in the same quantity could be made to strike a blue colour when the test was added.

Experiment 8.

The experiment was repeated upon a dog. In this I was assisted by Mr. BRODIE, Mr. WILLIAM BRANDE, Mr. CLIFT, and Mr. GATCOMBE. After the thoracic duct had been secured, two ounces of strong infusion of rhubarb were injected into the stomach, and in an hour the dog was killed. The urine in the bladder, on the addition of potash, became deeply tinged with rhubarb. The bile in the gall bladder, by a similar test, was found to contain rhubarb. The lacteal vessels in several parts of the mesentery had burst, and chyle was extravasated into the cellular membrane; the thoracic duct had given way in the lower part of the posterior mediastinum, and chyle was extravasated. Above the ruptured part the thoracic duct was much distended with chyle; it was readily traced to the ligature, by which it was completely secured.

These experiments appeared to establish the fact, that the thoracic duct was not the channel through which the infusion of rhubarb was conveyed to the circulation of the blood, and it now became easy to ascertain, whether it passed through the spleen, by extirpating that organ, and repeating the last experiment.

On the 21st of October, 1810, the following experiment, was made with the assistance of Mr. BRODIE, Mr. CLIFT, Mr. GATCOMBE, and Mr. MONEY.

Experiment 3.

The thoracic duct near its termination was secured in a dog, whose spleen had been removed four days before, and three ounces of infusion of rhubarb were injected into the stomach: in an hour and half the dog was killed, and the urine was found strongly impregnated with rhubarb; and on examination, the thoracic duct was found to be completely secured by the ligature. Several of the lacteals had burst, but the duct itself had not given way; it was greatly distended with chyle and lymph.

By this experiment it was completely ascertained, that the spleen is not the channel through which the infusion of rhubarb is conveyed into the circulation of the blood, as I had been led to believe, and therefore the rhubarb, in my former experiments detected in the spleen, must have been deposited there in the same manner as in the urine, and in the bile.

The detection of this error made me more anxious to avoid being misled respecting the thoracic duct; and therefore, although there was little probability that the infusion of rhubarb could have passed into the lymphatic vessels, which open into the blood vessels of the right side of the neck, I thought it right, before I proceeded further, to repeat the experiment, securing the termination of the thoracic duct on the left side, and the lymphatic trunk of the right side, where it empties itself into the angle between the jugular and subclavian vein. This was done on the 28th of October, 1810, with the assistance of the same persons as in the last experiment.

Experiment 4.

The thoracic duct of a dog was tied, as in the former experiment; in doing it the duct was wounded, and about a dram of chyle flowed out; the lymphatic trunk of the right side was then secured. After this, three ounces of infusion of rhubarb were injected into the stomach, and in an hour the dog was killed. The urine and the bile were found distinctly impregnated with rhubarb. On opening the thorax, some absorbent vessels, distended with lymph, were seen on the right side of the spine, entering an absorbent gland on the second dorsal vertebra, and the vasa efferentia from the gland were seen uniting with other absorbent vessels, and extending towards the right shoulder, where they formed a common trunk with the absorbents from the neck and axilla; this trunk was found included in the ligature. The thoracic duct was moderately distended with a mixture of chyle and lymph; in tracing it upwards, an opening was seen in it immediately below the ligature, through which the contents readily passed out when pressure was made on the duct: above this opening the duct was completely secured by the ligature. Nearly a dram of the fluid contained in the thoracic duct was collected and tested by potash, but there did not appear to be any impregnation of rhubarb.

Experiment 5.

The last experiment was repeated on another dog, on the 21st of January, 1811, with the assistance of Mr. Brodie, Mr. W. Brande, Mr. Clift, and Mr. Gatcombe. The dog was killed an hour after the thoracic duct and lymphatic trunk had been secured, and the infusion of rhubarb had been injected into the stomach.

In tying the right lymphatic trunk, a lymphatic vessel, from the thorax going to join it, was wounded, from which chyle flowed out in considerable quantity during the whole time of the experiment; a short time before the dog was killed some of it was collected, but on testing it with potash no rhubarb was detected in it.

The urine was found impregnated with rhubarb, as was also the bile from the gall bladder; but both in a less degree than in the last experiment. The lacteal vessels and mesenteric glands were much distended with chyle; and on cutting into the glands chyle flowed out in considerable quantity. Some of this was collected and tested with potash, but shewed no evidence of rhubarb being contained in it. The thoracic duct was much distended; it was traced to the ligature, and was found to be completely secured.

Lymphatic vessels from the right side of the posterior mediastinum, were seen extending towards the ligature that had been tied on that side; they were nearly empty; and the trunk formed by the junction of these with the lymphatic vessels from the right axilla, and from the right side of the neck, was seen distinctly included in the ligature.

While Mr. BRODIE was tracing the thoracic duct, Mr. WILLIAM BRANDE was making an infusion of the spleen, and shewed me a section of it, in which the cells were larger, and more distinct, than I had ever seen them in a dog. There was a slight tinge of rhubarb in the infusion from the spleen. A similar infusion was made of the liver; but the quantity of blood contained in it being much greater than in the spleen, the appearance was not sufficiently distinct to decide whether it contained rhubarb or not. These experiments appear com-

pletely to establish the fact, that the rhubarb did not pass through the thoracic duct, and therefore must have got into the circulation of the blood by some other channel. They likewise completely overturn the opinion I had adopted of the spleen being the medium by which the rhubarb had been conveyed, and show that the spleen answers some other purposes in the animal economy.

The rhubarb found in the spleen does not arrive there before it enters the circulation, it is therefore most probably afterwards deposited in the cells in the form of a secretion. That the rhubarb goes into the circulation is proved by my former experiments, in which it was detected in the splenic vein. The prussiate of potash is hardly to be discovered in the blood of a living animal, since the proportion which strikes a blue colour on the addition of solution of iron, is greater than the circulating fluids can be expected to contain at any one time, as it goes off by the secretions nearly as fast as it is received into the blood vessels. In a moderately sized ass more than two drams must be dissolved in the blood before its presence there can be detected.

That the fluid contained in the cells of the spleen is secreted there, is rendered highly probable, since it is most abundant while the digestive organs are employed, and scarcely at all met with when the animal has been sometime without food. The great objection to this opinion is, there being no excretory duct but the lymphatic vessels of the spleen; these however are both larger and more numerous than in any other organ; they are found in the ass to form one common trunk, which opens into a large gland on the side of the thoracic duct, just above the receptaculum chyli; and when the quick-

silver is made to pass through the branches of this gland, there is a trunk equally large on the opposite side, which makes an angle, and then terminates in the thoracic duct. This fact I ascertained at the Veterinary College, assisted by the Deputy Professor Mr. SEWELL, and Mr. CLIFT. These lymphatic vessels are equally large as the excretory ducts of any other glands, and therefore sufficient to carry off the secretion formed in the cells of the spleen; and where a secretion is to be carried into the thoracic duct, it would be a deviation from the general plan of the animal economy, were any but lymphatic vessels employed for that purpose.

It is a strong circumstance in favour of the secretion being so conveyed, that in the last experiment, the lacteals and cells of the spleen were unusually turgid, being placed under similar circumstances, the thoracic duct being so full as not to receive their contents.

The purposes that are answered by such a secretion from the spleen into the thoracic duct cannot at present be ascertained.

IX. *On the Composition of Zeolite.* By James Smithson, *Esq. F. R. S.*

Read February 7, 1811.

Mineral bodies being, in fact, *native chemical preparations*, perfectly analogous to those of the laboratory of art, it is only by chemical means, that their species can be ascertained with any degree of certainty, especially under all the variations of mechanical state and intimate admixture with each other, to which they are subject.

And accordingly, we see those methods which profess to supersede the necessity of chemistry in mineralogy, and to decide upon the species of it by other means than her's, yet bringing an unavoidable tribute of homage to her superior powers, by turning to her for a solution of the difficulties which continually arise to them, and to obtain firm grounds to relinquish or adopt the conclusions to which the principles they employ, lead them.

Zeolite and natrolite have been universally admitted to be species distinct from each other, from Mr. Klaproth having discovered a considerable quantity of soda and no lime, in the composition of the latter, while Mr. Vauquelin had not found any portion of either of the fixed alkalies, but a considerable one of lime, in his analysis of zeolite.*

The natrolite has been lately met with under a regular

* Journal des Mines, No. XLIV.

crystalline form, and this form appears to be perfectly similar to that of zeolite, but Mr. HAÜY has not judged himself warranted by this circumstance, to consider these two bodies as of the same species, because zeolite, he says, "does not contain an atom of soda." *

I had many years ago found soda in what I considered to be zeolites, which I had collected in the island of Staffa, having formed GLAUBER'S salt by treating them with sulphuric acid; and I have since repeatedly ascertained the presence of the same principle in similar stones from various other places; and Dr. HUTTON and Dr. KENNEDY, had likewise detected soda in bodies, to which they gave the name of zeolite.

There was, however, no certainty that the subjects of any of these experiments were of the same nature as what Mr. VAUQUELIN had examined, were of that species which Mr. HAÜY calls mesotype.

Mr. HAÜY was so obliging as to send me lately, some specimens of minerals. There happened to be amongst them a cluster of zeolite in rectangular tetrahedral prisms, terminated by obtuse tetrahedral pyramids whose faces coincided with those of the prism. These crystals were of a considerable size, and perfectly homogeneous, and labelled by himself "*Mesotype pyramidée du départ. du Puy de Dôme.*" I availed myself of this very favourable opportunity, to ascertain whether the mesotype of Mr. HAÜY and natrolite, did or did not differ in their composition, and the results of the experiments have been entirely unfavourable to their separation, as the following account of them will show.

10 grains of this zeolite being kept red hot for five minutes

* Journal des Mines, No. CL. Juin 1810, p. 458.

lost 0.75 grains, and became opaque and friable. In a second experiment, 10 grains being exposed for 10 minutes to a stronger fire, lost 0.95 grains, and consolidated into a hard transparent state.

10 grains of this zeolite, which had not been heated, were reduced to a fine powder, and diluted muriatic acid poured upon it. On standing some hours, without any application of heat, the zeolite entirely dissolved, and some hours after, the solution became a jelly: this jelly was evaporated to a dry state, and then made red hot.

Water was repeatedly poured on to this ignited matter till nothing more could be extracted from it. This solution was gently evaporated to a dry state, and this residuum made slightly red hot. It then weighed 3.15 grains. It was *muriate of soda.*

The solution of this muriate of soda being tried with solutions of carbonate of ammonia and oxalic acid, did not afford the least precipitate, which would have happened had the zeolite contained any lime, as the muriate of lime* would not have been decomposed by the ignition.

The remaining matter, from which this muriate of soda had been extracted, was repeatedly digested with marine acid, till all that was soluble was dissolved. What remained was silica, and, after being made red hot, weighed 4.9 grains.

The muriatic solution, which had been decanted off from the silica, was exhaled to a dry state, and the matter left made red hot. It was alumina.

* These names are retained for the present, as being familiar, though, since Mr. Davy's important discovery of the nature of what was called oxymuriatic acid, the substances to which they are applied, are known not to be salts, but metallic compounds analogous to oxides.

To discover whether any magnesia was contained amongst this alumina, it was dissolved in sulphuric acid, the solution evaporated to a dry state, and ignited. Water did extract some saline matter from this ignited alumina, but it had not at all the appearance of sulphate of magnesia, and proved to be some sulphate of alumina which had escaped decomposition, for on an addition of sulphate of ammonia to it, it produced crystals of compound sulphate of alumina and ammonia, in regular octahedrons.

This alum and alumina were again mixed and digested in ammonia, and the whole dried and made red hot. The alumina left, weighed 3.1 grains.

Being suspected to contain still some sulphuric acid, this alumina was dissolved in nitric acid, and an excess of acetate of barytes added. A precipitate of sulphate of barytes fell, which after being edulcorated and made red hot, weighed 1.2 grains. If we admit $\frac{1}{3}$ of sulphate of barytes to be sulphuric acid, the quantity of the alumina will be $= 3.1 - 0.4 = 2.7$ grains.

From the experiments of Dr. MARCET,* it appears that 3.15 grains of muriate of soda, afford 1.7 grains of soda.

Hence, according to the foregoing experiments, the 10 grains of zeolite analysed, consisted of

Silica - - -	4.90
Alumina - -	2.70
Soda - - -	1.70
Ice - - -	0.95
	10.25

* Phil. Trans. 1807.

As these experiments had been undertaken more for the purpose of ascertaining the nature of the component parts of this zeolite than their proportions, the object of them was considered as accomplished, although perfect accuracy in the latter respect, had not been attained, and which, indeed, the analysis we possess of natrolite by the illustrious chemist of Berlin, renders unnecessary.

I am induced to prefer the name of zeolite for this species of stone, to any other name, from an unwillingness to obliterate entirely from the nomenclature of mineralogy, while arbitrary names are retained in it, all trace of one of the discoveries of the greatest mineralogist who has yet appeared, and which, at the time it was made, was considered as, and was, a very considerable one, being the first addition of an earthy species, made by scientific means, to those established immemorially by miners and lapidaries, and hence having, with tungstein and nickel, led the way to the great and brilliant extension which mineralogy has since received. And, of the several substances, which, from the state of science in his time, certain common qualities induced Baron CRONSTEDT to associate together under the name of zeolite; it is this which has been most immediately understood as such, and whose qualities have been assumed as the characteristic ones of the species.

Indeed, I think that the name imposed on a substance by the discoverer of it, ought to be held in some degree sacred, and not altered without the most urgent necessity for doing it. It is but a feeble and just retribution of respect for the service which he has rendered to science.

Professor STRUVE, of Lausanne, whose skill in mineralogy

is well known, having mentioned to me, in one of his letters, that from some experiments of his own, he was led to suspect the existence of phosphoric acid in several stones, and particularly in the zeolite of Auvergne, I have directed my enquiries to this point, but have not found the phosphoric, or any other acknowledged mineral acid, in this zeolite.

Many persons, from experiencing much difficulty in comprehending the combination together of the earths, have been led to suppose the existence of undiscovered acids in stony crystals. If quartz be itself considered as an acid, to which order of bodies its qualities much more nearly assimilate it, than to the earths, their composition becomes readily intelligible. They will then be neutral salts, silicates, either simple or compound. Zeolite will be a compound salt, a hydrated silicate of alumina and soda, and hence a compound of alumina not very dissimilar to alum. And topaz, whose singular ingredients, discovered by Mr. KLAPROTH, have called forth a query from the celebrated Mr. VAUQUELIN, with regard to the mode of their existence together,* will be likewise a compound salt, consisting of silicate of alumina, and fluate of alumina.

Our acquaintance with the composition of the several mineral substances, is yet far too inaccurate to render it possible to point out with any degree of certainty, the one of which zeolite is an hydrate, however the agreement of the two substances in the nature of their constituent parts, and in their being both electrical by heat, directs conjecture towards tourmaline.

St. James's Place, Jan. 22, 1811.

* Annales du Museum d'Hist. Nat. tome 6, p. 24.

Addition to the Account of native Minium.

After I had communicated to the President, the account of the discovery of native minium, printed in the Philosophical Transactions for 1806, I learned that this ore came from the lead mines of Breylau in Westphalia.

X. *Experiments and Observations on the different Modes in which Death is produced by certain vegetable Poisons.* By B. C. Brodie, *Esq. F. R. S. Communicated by the Society for promoting the Knowledge of Animal Chemistry.*

Read February 21, 1811.

I. The following experiments were instituted with a view to ascertain, in what manner certain substances act on the animal system, so as to occasion death, independently of mechanical injury. I was led to the inquiry, from the subject of it appearing to be of considerable interest and importance, and from a hope, that, in the present improved state of physiological knowledge, we might be enabled to arrive at some more satisfactory conclusions, than had been deduced from any former observations.

The substances, which act as poisons when applied to the animal body are very numerous. In the experiments, which I have hitherto made, I have employed vegetable poisons only. Of these I have selected such, as are very active and certain in producing their effects, believing that, on this account, the exact nature of those effects would be more readily ascertained. The principal objects, which I have kept in view have been to determine, on which of the vital organs the poison employed exercises its primary influence, and through what medium that organ becomes affected. I have also endeavoured

to ascertain by what means the fatal consequences of some poisons may be prevented. With some of the conclusions, which I have ventured to draw, so far as I know, we were not before acquainted; and others of them, though not entirely new, had not been previously established by satisfactory experiments.

I shall relate first those experiments, in which poisons were applied internally, that is to the mucous membranes of the tongue or alimentary canal, and afterwards those, in which poisons were applied to wounded surfaces.

II. *Experiments with Poisons applied to the Tongue or alimentary Canal.*

Experiments with Alcohol.

When spirits are taken into the stomach, in a certain quantity, they produce that kind of delirium, which, constitutes intoxication: when taken in a larger quantity, it is well known that, they destroy life altogether, and that in the course of a very short space of time. Intoxication is a derangement of the functions of the mind, and, as these are in some way connected with those of the brain, it seems probable, that it is by acting on this organ, that spirits when taken into the stomach occasion death. In order to ascertain how far this conclusion is just, I made the following experiments.*

Experiment 1. I poured two drams of proof spirits down

* I am indebted to Dr. E. N. BANCROFT for his assistance in many of the experiments which I am about to detail. Mr. W. BRANDE lent me his assistance in the greater part of those which were made. I have been further assisted by Mr. BROUGHTON, Mr. R. RAWLINS, and Mr. R. GATCOMBE, and by several other gentlemen.

the œsophagus of a cat. Instantly he struggled violently; then lay on one side, perfectly motionless and insensible; the breathing was laboured and stertorous, and the pulsations of the heart were very frequent. He continued in this state for seven or eight minutes; then began to recover; the respirations became easier, and presently he stood up, and was able to walk.

Exp. 2. I injected an ounce and a half of proof spirits into the stomach of a large full-grown rabbit, by means of an elastic gum tube passed down the œsophagus. The same symptoms took place as in the last experiment; but the animal did not begin to recover from the state of insensibility, until forty minutes had elapsed from the time of the injection.

Exp. 3. Seven drams of proof spirits were injected into the stomach of a younger rabbit. Two minutes afterwards, he evidently was affected by the spirits, and in three minutes more he lay on one side motionless and insensible. The pupils of the eyes were perfectly dilated; there were occasional slight convulsive motions of the extremities; the respiration was laborious, it was gradually performed at longer and longer intervals, and at the end of an hour and fifteen minutes had entirely ceased. Two minutes after the animal was apparently dead, I opened into the thorax, and found the heart acting with moderate force and frequency, circulating dark coloured blood. I introduced a tube into the trachea, and produced artificial respiration by inflating the lungs, and found that by these means the action of the heart might be kept up to the natural standard, as in an animal from whom the head is removed.

Exp. 4. I injected into the stomach of a rabbit two ounces of proof spirits. The injection was scarcely completed, when the animal became perfectly insensible. Precisely the same symptoms took place as in the last experiment, and at the end of twenty-seven minutes, from the time of the injection, the rabbit was apparently dead; but on examining the thorax the heart was found still acting, as in the last experiment.

It has been shewn by M. Bichat, and the observation has been confirmed by some experiments, which I have lately had the honour of communicating to this learned Society, that the brain is not directly necessary to the action of the heart, and that, when the functions of the brain are destroyed, the heart continues to contract for some time afterwards, and then ceases only in consequence of the suspension of respiration, which is under the influence of the brain.

It would appear, from the experiments, which I have just detailed, that the symptoms produced by a large quantity of spirits taken into the stomach, arisé entirely from disturbance of the functions of the brain. The complete insensibility to external impressions; the dilatation of the pupils of the eyes; and the loss of motion, indicate that the functions of this organ are suspended; respiration, which is under its influence, is ill performed, and at last altogether ceases; while the heart, to the action of which the brain is not directly necessary, continues to contract, circulating dark coloured blood for some time afterwards.

There is a striking analogy between the symptoms arising from spirits taken internally, and those produced by injuries of the brain.

Concussion of the brain, which may be considered as the

slightest degree of injury, occasions a state of mind resembling intoxication, and the resemblance in some instances is so complete, that the most accurate observer cannot form a diagnosis, except from the history of the case. Pressure on the brain, which is a more severe injury than concussion, produces loss of motion, insensibility, dilatation of the pupils; respiration becomes laboured and stertorous, is performed at long intervals, and at last altogether ceases, and the patient dies.

It forms an interesting matter of inquiry, whether spirits when taken into the stomach produce their effects on the brain, by being absorbed into the circulation, or in consequence of the sympathy, that exists between these organs by means of the nerves. The following circumstances lead me to conclude that they act in the last of these two ways.

1. In experiments where animals have been killed by the injection of spirits into the stomach, I have found this organ to bear the marks of great inflammation, but never found any preternatural appearances whatever in the brain. 2. The effects of spirits taken into the stomach in the last experiment were so instantaneous, that it appears impossible that absorption should have taken place before they were produced. 3. A person who is intoxicated, frequently becomes suddenly sober after vomiting. 4. In the experiments, which I have just related, I mixed tincture of rhubarb with the spirits, knowing from the experiments of Mr. HOME and Mr. WILLIAM BRANDE, that this, when absorbed into the circulation, was readily separated from the blood by the kidneys, and that very small quantities might be detected in the urine by the addition of potash; but, though I never failed to find urine in the bladder, I never detected rhubarb in it.

The including the termination of the thoracic duct in a ligature does not prevent spirits, when taken into the stomach, from producing their usual effects on the nervous system, but subsequent observations, which Mr. HOME has already communicated to this Society, have shewn that no conclusion can be drawn from this experiment.

That a poison may affect a distant organ, through the medium of the nerves, without entering the circulation, is proved by the well-known circumstance of solution of the extract of *Belladonna*, when applied to the tunica conjunctiva of the eye, occasioning dilatation of the pupil of the same eye, though no other part of the system is affected.

It has been formerly supposed by Dr. MEAD and other physiologists, that a poison may produce death by acting on the extremities of the nerves of the stomach and intestines, without being absorbed into the circulation. That it should by these means be capable of affecting the brain is not to be wondered at, when we consider the numerous and various sympathies between this organ and the alimentary canal, evidently independent of any other communication than the nerves.

*Experiments with the Essential Oil of Bitter Almonds.**

Experiment 5. One drop of the essential oil of bitter almonds was applied to the tongue of a young cat. She was instantly seized with violent convulsions; then lay on one side motionless, insensible, breathing in a hurried manner; the respirations

* The essential oil of bitter almonds does not appear to differ from the essential oil of laurel. I was furnished with a quantity of it, first by my friend Mr. WILLIAM BRANDE, and afterwards by Mr. COCKS of Southampton-street.

became laboured, took place at longer and longer intervals, and at the end of five minutes, from the application of the poison, had entirely ceased, and the animal was apparently dead; but, on opening the thorax, the heart was found acting regularly eighty times in a minute, circulating dark coloured blood, and it continued to act for six or seven minutes afterwards.

Exp. 6. I injected into the rectum of a cat half an ounce of water, with two drops of the essential oil. In two minutes afterwards, he was affected with symptoms similar to those, which occurred in the last experiment, and at the end of five minutes, from the injection of the poison, he was apparently dead. Two minutes after apparent death, the heart was found acting eighty times in a minute. On dissection, no preternatural appearances were found either in the internal membrane of the rectum, or the brain.

The symptoms produced by this poison, and the circumstance of the heart continuing to contract after apparent death, lead to the conclusion that it occasions death by disturbing the functions of the brain.

While engaged in these last experiments, I dipped the blunt end of a probe into the essential oil, and applied it to my tongue, meaning to taste it, and having no suspicion that so small a quantity could produce any of its specific effects on the nervous system; but scarcely had I applied it, when I experienced a very remarkable and unpleasant sensation, which I referred chiefly to the epigastric region, but the exact nature of which I cannot describe, because I know nothing precisely similar to it. At the same time there was a sense of weakness in my limbs, as if I had not the command of my

muscles, and I thought that I was about to fall. However, these sensations were momentary, and I experienced no inconvenience whatever afterwards.

I afterwards applied a more minute quantity of the essential oil to my tongue several times, without experiencing from it any disagreeable effects; but on applying a larger quantity, I was affected with the same momentary sensations as in the former instance, and there was a recurrence of them in three or four seconds after the first attack had subsided.

From the instantaneousness, with which the effects are produced; and from its acting more speedily when applied to the tongue, than when injected into the intestine, though the latter presents a better absorbing surface, we may conclude that this poison acts on the brain through the medium of the nerves, without being absorbed into the circulation.

Experiment with the Juice of the Leaves of Aconite.

Exp. 7. An ounce of this juice was injected into the rectum of a cat. Three minutes afterwards he voided what appeared to be nearly the whole of the injection; he then stood for some minutes perfectly motionless, with his legs drawn together; at the end of nine minutes, from the time of the injection, he retched and vomited; then attempted to walk, but faultered and fell at every step, as if from giddiness. At the end of thirteen minutes, he lay on one side insensible, motionless, except some slight convulsive motions of the limbs. The respiration became slow and laboured; and at forty-seven minutes from the time of the injection, he was apparently dead. One minute and a half afterwards, the heart was found contracting regularly one hundred times in a minute.

It appears from this experiment, that the juice of aconite, when injected into the intestine, occasions death by destroying the functions of the brain. From the analogy of other poisons, it is rendered probable that it acts on the brain through the medium of the nerves, without being absorbed into the circulation. This opinion is confirmed by the following circumstance: if a small quantity of the leaf of aconite is chewed, it occasions a remarkable sense of numbness of the lips and gums, which does not subside for two or three hours.

Experiments with the Infusion of Tobacco.

Exp. 8. Four ounces of infusion of tobacco were injected into the rectum of a dog. Four minutes afterwards he retched, but did not vomit; he then became faint, and lay motionless on one side; at the end of nine minutes from the time of the injection, the heart could not be felt; he gasped for breath at long intervals; and in another minute there was no appearance whatever of life. I immediately laid open the cavities of the thorax and abdomen. The heart was much distended, and had entirely ceased to contract; there was no peristaltic motion of the intestines.

Exp. 9. An ounce of very strong infusion of tobacco was injected into the rectum of a cat. Symptoms were produced similar to those, which occurred in the last experiment, and the animal died at the end of seven minutes from the time of the injection. On opening the thorax immediately after death, the heart was found extremely distended, and to have entirely ceased acting, with the exception of a slight tremulous motion of the auricles.

Exp. 10. Three ounces of infusion of tobacco were injected into the rectum of a dog. He was affected with symptoms similar to those in the former experiments, and died at the end of ten minutes. On opening the thorax immediately after death, I found the heart much distended, and to have entirely ceased contracting.

Exp. 11. Three ounces of infusion of tobacco were injected into the rectum of a dog. Immediately there took place tremulous contractions of the voluntary muscles. Five minutes afterwards the injection was repeated in the same quantity. The dog then was sick, and threw up some of the infusion, with other matter, from the stomach; he became faint, and died ten minutes after the second injection. Immediately after respiration had ceased, I opened the thorax, and found the heart extremely distended, and without any evident contraction, except of the appendix of the right auricle, which every now and then contracted in a slight degree. I divided the pericardium on the right side. In consequence of the extreme distension of the heart, this could not be done without irritating the fibres with the point of the scalpel. Immediately both auricles and ventricles began to contract with considerable force, so as to restore the circulation. Artificial respiration was produced, and the circulation was kept up for more than half an hour, beyond which time the experiment was not continued.

We may conclude from these experiments, that the effect of the infusion of tobacco, when injected into the intestine of a living animal, is to destroy the action of the heart, stopping the circulation and producing syncope. It appeared to me that the action of the heart ceased even before the animal had

ceased to respire; and this was confirmed by another experiment, in which, in a dog killed by the infusion of tobacco, I found the cavities of the left side of the heart to contain scarlet blood, while in those of the right side the blood was dark coloured. This poison therefore differs materially from alcohol, the essential oil of almonds, and the juice of aconite, which have no direct influence on the action of the heart. The infusion of tobacco renders the heart insensible to the stimulus of the blood, but it does not altogether destroy the power of muscular contraction, since the heart resumed its action in one instance on the division of the pericardium, and I have found that the voluntary muscles of an animal killed by this poison, are as readily stimulated to contract by the influence of the VOLTAIC battery, as if it had been killed in any other manner. At the same time, however, that the infusion of tobacco destroys the action of the heart, it appears to destroy also the functions of the brain, since these did not return in the last experiment; although the circulation was restored, and kept up by artificial respiration.

Since there is no direct communication between the intestinal canal and the heart, I was at first induced to suppose that the latter becomes affected in consequence of the infusion being conveyed into the blood by absorption. Some circumstances in the following experiment have since led me to doubt whether this is the case.

Exp. 12. In a dog, whose head was removed, I kept up the circulation by means of artificial respiration, in the manner already described in the account of some experiments, which I lately communicated to this Society. I then injected into the stomach and intestines nine ounces of infusion of tobacco,

At the time of the injection, the body of the animal lay perfectly quiet and motionless on the table; the heart acted regularly one hundred times in a minute. Ten minutes afterwards the pulse rose to one hundred and forty in a minute; the peristaltic motion of the intestines was much increased, and the voluntary muscles in every part of the body were thrown into repeated and violent spasmodic action. The joints of the extremities were alternately bent and extended; the muscles of the spine, abdomen, and tail alternately relaxed and contracted, so as to turn the whole animal from one side to the other. I have observed, in other instances, spasmodic actions of the muscles, where the circulation was kept up by artificial respiration, after the removal of the head, but not at all to be compared, either in strength or frequency, with those, which took place on this occasion. I made pressure on the abdominal aorta for more than a minute, so as to obstruct the circulation of the blood in the lower extremities; but the muscular contractions were not lessened in consequence. Half an hour after the injection of the infusion, the artificial respiration was discontinued. The heart continued to act, circulating dark coloured blood; the muscular contractions continued, but gradually diminished in strength and frequency. I tied a ligature round the vessels at the base of the heart, so as to stop the circulation, nevertheless the muscular contractions still continued, though less frequent and forcible than before, and some minutes elapsed before they entirely ceased.

In this experiment, the disposition to contraction in the muscles was very much increased, instead of being diminished, as in those just related. If the infusion of tobacco influences the heart from being absorbed into the blood, and thus coming

into actual contact with its fibres, there is no evident reason why, the removal of the brain, and the employment of artificial respiration, should occasion so material a difference in its effects. If the contractions of the voluntary muscles had depended on the infusion circulating with the blood, it is reasonable to suppose that the pressure on the aorta would have occasioned some diminution of them, and that the complete obstruction of the circulation would have caused them to cease altogether.

From these considerations, I am induced, on the whole, to believe that the infusion of tobacco, when injected into the intestines, influences the heart through the medium of the nervous system; but I have not been able to devise any experiment, by which the truth or fallacy of this opinion might be put beyond the reach of doubt.

It appears remarkable, that the brain and nervous system, although not necessary to the action of the heart, should, when under the influence of the infusion of tobacco, be capable of influencing this organ so as to stop its action; but this is analogous to what we see occur in consequence of violent emotions of the mind. Those states of the nervous system, which accompany the passions of joy, fear, or anger, when existing in a moderate degree, render the heart more sensible to the stimulus of the blood, and increase the frequency of its contractions; while, when the same passions exist in a greater degree, the heart is rendered altogether insensible to the stimulus of the blood, and syncope ensues.

*Experiments with the Empyreumatic Oil of Tobacco.**

Exp. 13. Less than a drop of this oil was applied to the tongue of a young cat. Instantly violent convulsions took place in all the muscles, and the respirations became very frequent. In five minutes after the application, she lay on one side insensible, with slight spasmodic actions of the muscles. At the end of eleven minutes, she retched, but did not vomit. In a quarter of an hour, she appeared to be recovering. I repeated the application of the poison, and she was again seized with violent convulsions, and became insensible, breathing at long intervals, and in two minutes from the second application respiration had entirely ceased, and she was apparently dead. On opening the thorax, I found the heart acting with regularity and strength, circulating dark-coloured blood. I introduced a tube into the trachea, and produced artificial respiration; the contractions of the heart became augmented in force and frequency, and there was no evident diminution in six or seven minutes, during which the artificial respiration was continued.

On dissection, nothing remarkable was found in the appearance of the tongue or brain.

The symptoms and mode of death, in this experiment, did not essentially differ from those produced by the essential oil of almonds. I was surprised to find the effects of the empyreumatic oil so entirely different from those of the infusion of

* I was furnished with the empyreumatic oil of tobacco by Mr. W. Brande. It may be procured by subjecting the leaves of tobacco to distillation in a heat above that of boiling water: a quantity of watery fluid comes over, on the surface of which is a thin film of unctuous substance.

tobacco. Supposing that this difference might arise from the poison being more concentrated in the oil than in the infusion, I made the following experiments.

Exp. 14. A drop of the oil of tobacco was suspended in an ounce and a half of water by means of mucilage of gum arabic, and the whole was injected into the rectum of a dog. In two minutes afterwards he became faint, retched, but did not vomit. He appeared to be recovering from this state, and in twenty-five minutes after the first injection, it was repeated in the same quantity. He was then seized with symptoms similar to those in the last experiment, and in two minutes and a half he was apparently dead.

Two minutes after apparent death, on the thorax being opened into, the heart was found acting, regularly one hundred times in a minute, and it continued acting for several minutes.

Exp. 15. A drop of the empyreumatic oil of tobacco with an ounce of water was injected into the rectum of a cat. The symptoms produced were in essential circumstances similar to those, which occurred in the last experiment. The animal was apparently dead in five minutes after the injection, and the heart continued to contract for several minutes afterwards.

We may conclude from these experiments, that the empyreumatic oil of tobacco, whether applied to the tongue, or injected into the intestine, does not stop the action of the heart and induce syncope, like the infusion of tobacco; but that it occasions death by destroying the functions of the brain, without directly acting on the circulation. In other words, its effects are similar to those of alcohol, the juice of aconite, and the essential oil of almonds.

III. *Experiments with Poisons applied to wounded Surfaces.*

Experiments with the Essential Oil of Almonds.

Exp. 16. I made an incision in the thigh of a rabbit, and introduced two drops of essential oil between the skin and the muscles. In four minutes after the application, he was seized with violent convulsions, and became insensible, and in two minutes more he was apparently dead; but the heart was felt through the ribs acting one hundred and twenty times in a minute, and it continued acting for several minutes. There were no other appearances in the limb, than would have resulted from an ordinary wound.

Exp. 17. Two drops of the essential oil of almonds were introduced into a wound in the side of a mouse. Two minutes afterwards he was affected with symptoms similar to those which occurred in the last experiment, and in two minutes more he was apparently dead, but the heart continued to contract for some minutes afterwards.

From the experiments, which I have just related, and from others which it appears unnecessary to detail, as the general results were the same, I have learned that where the essential oil of almonds is applied to a wound, its effects are not so instantaneous as when it is applied to the tongue; otherwise there is no difference in its effects in whatever manner it is applied.

Experiment with the Juice of the Leaves of Aconite.

Exp. 18. I made a wound in the side of a young rabbit, and introduced, between the skin and muscles, about twenty

drops of the juice of aconite. Twenty-three minutes afterwards he was affected with symptoms in all essential respects similar to those, which occurred in an experiment already related, where the juice was injected into the rectum, and at the end of forty-seven minutes from the application of the poison, he was apparently dead. Two minutes after apparent death, the heart was found contracting, but very feebly.

*Experiments with the Woorara.**

Exp. 19. A small quantity of the woorara in powder was applied to a wound in the side of a Guinea pig. Ten minutes afterwards the animal was unable to walk; then he became quite motionless, except some slight occasional convulsions. He gradually became insensible, the respirations were laboured, and at the end of fourteen minutes from the application of the poison, the respiration had entirely ceased, and he was apparently dead; but on opening the thorax, the heart was found acting seventy times in a minute, circulating dark coloured blood, and it continued to contract for several minutes afterwards. On dissection no preternatural appearances were observed in the brain; nor was there any other appearance in the limb than would have arisen from an ordinary wound.

Exp. 20. I made a wound in the side of a Guinea pig, and introduced into it about two grains of the woorara in powder.

* The Woorara is a poison with which the Indians of Guiana arm the points of their arrows. It appears not to differ essentially from the Ticunas, which was employed in the experiments of the Abbé FONTANA. I am indebted to Dr. E. N. BANCROFT, who not only furnished me with some of the Woorara which he had in his possession, but also lent me his assistance in the experiments which were made with it.

At the end of twenty-five minutes, symptoms took place very similar to those, which occurred in the last experiment, and in thirteen minutes more the animal was apparently dead; but the heart continued to contract one hundred and eight times in a minute, and by means of artificial respiration the circulation was kept up for more than twenty minutes.

The results of other experiments, which I have made with the woorara, were similar to those just described. The heart continued to act after apparent death, and the circulation might be kept up by means of artificial respiration. It is evident that this poison acts in some way or another on the brain, and that the cessation of the functions of this organ is the immediate cause of death.

I found in these experiments, that the best mode of applying the woorara is when it is dissolved in water to the consistence of a thin paste. I first made the wound, and then smeared the poison over it with the end of the scalpel. I found that the animal was more speedily and certainly affected, if there was some hæmorrhage, unless the hæmorrhage was very copious, when it produced an opposite effect, by washing the poison away from the wound. When the poison was applied in large quantity, it sometimes began to act in six or seven minutes. Never more than half an hour elapsed from the time of the poison being inserted, to that of the animal being affected, except in one instance, where a ligature was applied on the limb, which will be mentioned afterwards. The woorara, which I employed, had been preserved for some years, which will account for its having been less active, than it has been described to be by those, who had witnessed its effects when in a recent state.

*Experiments with the Upas Antiar.**

Exp. 21. About two grains of this poison were made into a thin paste with water, and inserted into a wound in the thigh of a dog. Twelve minutes afterwards he became languid; at the end of fifteen minutes, the heart was found to beat very irregularly, and with frequent intermissions; after this, he had a slight rigor. At the end of twenty minutes, the heart beat very feebly and irregularly; he was languid; was sick and vomited; but the respirations were as frequent and as full as under natural circumstances, and he was perfectly sensible. At the end of twenty minutes, he suddenly fell on one side, and was apparently dead. I immediately opened into the thorax, and found the heart distended with blood in a very remarkable degree, and to have entirely ceased contracting. There was one distinct and full inspiration after I had begun making the incision into the thorax. The cavities of the left side of the heart contained scarlet blood, and those of the right side contained dark coloured blood, as in a living animal.

Exp. 22. A small quantity of the upas antiar, prepared as before, was inserted into a wound in the thigh of a young cat. She appeared languid in two minutes after the poison was inserted. The symptoms, which took place did not essentially differ from those which occurred in the last experiment, except that there were some convulsive motions of the limbs.

* We are informed, that the island of Java produces two powerful vegetable poisons, to one of which the natives give the name of *Upas tieuté*, and to the other that of *Upas antiar*. I was supplied with a quantity of the latter through the kindness of Mr. MARSDEN, who had some of it in his possession.

At eight minutes after the poison was inserted, she lay on one side, motionless and insensible, the heart could not be felt, but the respiration had not entirely ceased. On opening into the thorax, I found the heart to have ceased contracting. It was much distended with blood: and the blood in the cavities of the left side was of a scarlet colour. There were two full inspirations after the incision of the thorax was begun. On irritating the heart with the point of the scalpel, slight contractions took place in the fibres of the appendices of the auricles, but none in any other part.

Exp. 23. The experiment was repeated on a rabbit. The symptoms produced were similar to those in the last experiment; but the animal did not vomit, and the convulsive motions were in a less degree: he died eleven minutes after the poison was inserted. On opening the chest, the heart was found to have entirely ceased contracting; it was much distended with blood; and the blood in the cavities of the left side was of a scarlet colour. On irritating the heart with the point of the scalpel, the ventricles contracted, but not sufficiently to restore the circulation.

Exp. 24. About a grain of the upas antiar was inserted into a wound in the side of rabbit. He was affected with symptoms similar to those before described, and died in ten minutes after the poison was applied. On opening the thorax immediately after death, the heart was found to have ceased contracting, and the blood in the cavities of the left side was of a scarlet colour.

It appears from these experiments, that the upas antiar, when inserted into a wound, produces death (as infusion of tobacco does when injected into the intestine) by rendering the heart

insensible to the stimulus of the blood, and stopping the circulation. The heart beats feebly and irregularly before either the functions of the mind, or the respiration appear to be affected. Respiration is performed even after the circulation has ceased; and the left side of the heart is found after death to contain scarlet blood, which never can be the case, where the cause of death is the cessation of the functions of the brain or lungs. The convulsions, which occur when the circulation has nearly ceased, probably arise from the diminution of the supply of blood to the brain, resembling those, which take place in a person, who is dying from hæmorrhage.

There remains an interesting subject of inquiry, "through what medium do poisons influence the brain when applied to wounds?" That poisons applied in this manner do not produce their effects precisely in the same way as poisons taken internally, is rendered probable by this circumstance; that, some poisons, which are very powerful when applied to wounds even in small quantities, are either altogether inefficient when taken internally, or require to be given in very large quantities, in order to produce their effect, and vice versâ.

A poison applied to a wounded surface may be supposed to act on the brain in one of three ways,

1. By means of the nerves, like poisons taken internally.

2. By passing into the circulation through the absorbent vessels.

3. By passing directly into the circulation through the divided veins.

Exp. 25. In order to ascertain whether the woorara acts through the medium of the nerves, I exposed the axilla of a rabbit, and divided the spinal nerves supplying the upper

extremity, just before they unite to form the axillary plexus. The operation was performed with the greatest care. I not only divided every nervous filament, however small, which I could detect, but every portion of cellular membrane in the axilla, so that the artery and vein were left entirely insulated. I then made two wounds in the fore-arm, and inserted into them some of the woorara formed into a paste. Fourteen minutes after the poison was applied, the hind legs became paralytic, and in ten minutes more he died, with symptoms precisely similar to those, which took place in the former experiments, and the heart continued to act after apparent death. On dissection, the nerves of the upper extremity were particularly examined, but not the smallest filament could be found undivided.

I made the following experiment to ascertain whether the woorara passes into the circulation through the absorbent vessels.

Exp. 26. I tied a ligature round the thoracic duct of a dog, just before it perforates the angle of the left subclavian and jugular veins. I then made two wounds in the left hind leg, and introduced some of the woorara in powder into them. In less than a quarter of an hour he became affected with the usual symptoms, and died in a few minutes afterwards.

After death, I dissected the thoracic duct with great care. I found it to have been perfectly secured by the ligature. It was very much distended with chyle, and about two inches below its termination its coats had given way, and chyle was extravasated into the cellular membrane. The lymphatic vessels in the left axilla were distended in a very remarkable degree, and on dividing them, not less than a dram of lymph issued from the divided ends.

Since neither the division of the nerves, nor the obstruction of the thoracic duct interfere in the slightest degree with the effects of the woorara, there is presumptive evidence that it acts on the brain by entering the circulation through the divided veins. I endeavoured to ascertain, by experiment, whether this is really the case.

To apply ligatures to the large vessels of a limb only would evidently lead to no satisfactory conclusion, since the anastomosing vessels might still carry on the circulation. The only way, which I could devise of performing the experiment, was to include all the vessels, small as well as large, in a ligature.

Exp. 27. In order to make the experiment more satisfactorily, I exposed the sciatic nerve of a rabbit in the upper and posterior part of the thigh, and passed under it a tape half an inch wide. I then made a wound in the leg, and having introduced into it some of the woorara mixed with water, I tied the tape moderately tight on the fore-part of the thigh. Thus I interrupted the communication between the wounds and the other parts of the body, by means of the vessels, while that by means of the nerve still remained. After the ligature was tightened, I applied the woorara a second time, in another part of the leg. The rabbit was not at all affected, and at the end of an hour I removed the ligature. Being engaged in some other pursuit, I did not watch the animal so closely as I should otherwise have done; but twenty minutes after the ligature was removed, I found him lying on one side, motionless and insensible, evidently under the influence of the poison, but the symptoms were less violent than in most instances, and after lying in this state he recovered, and the limb became perfectly warm, and he regained the power of using it.

Exp. 28. I repeated the last experiment with this difference, that after having applied the poison, I made the ligature as tight as I could draw it. I removed the ligature at the end of an hour and twenty minutes, but the animal was not at all affected either before or after the removal of the ligature, and on the following day he had recovered the use of the limb.

Exp. 29. I repeated the experiment a third time, drawing the ligature very tight. At the end of forty-five minutes, the animal continued perfectly well, and the ligature was removed. I watched him for three quarters of an hour afterwards, but there were no symptoms of his being affected by the poison. On the following day the rabbit died, but this I attribute to the injury done to the limb and sciatic nerve by the ligature, as there was the appearance of inflammation in the parts in the neighbourhood of the ligature.

These three experiments were made with the greatest care. From the mode, in which the poison was applied, from the quantity employed, and from my prior experience, I should have entertained not the smallest doubt of the poison taking effect in every instance in less than twenty minutes, if no ligature had been applied. In two of the three, the quantity of woorara was more than had been used in any former experiments.

I have not judged it necessary to make any more experiments, with the ligature on the limb, because the numerous experiments of the Abbé FONTANA on the ticunas, coincide in their results, with those, which have just been detailed, and fully establish the efficacy of the ligature, in preventing the action of the poison. It is not to be wondered at, that the ligature should sometimes fail in its effects, since these must

evidently depend on the degree, in which the circulation is obstructed, and on the length of time during which the obstruction is continued.

There can be little doubt that the woorara affects the brain, by passing into the circulation through the divided vessels. It is probable that it does not produce its effects, until it enters the substance of the brain, along with the blood, in which it is dissolved; nor will the experiments of the Abbé FONTANA, in which he found the ticunas produce almost instant death when injected into the jugular vein of a rabbit, be found to militate against this conclusion, when we consider how short is the distance, which, in so small an animal, the blood has to pass from the jugular vein to the carotid artery, and the great rapidity of the circulation; since in a rabbit under the influence of terror, during such an experiment, the heart cannot be supposed to act so seldom as three times in a second.

I have made no experiments to ascertain through what medium other poisons when applied to wounds affect the vital organs, but from analogy we may suppose that they enter the circulation through the divided blood-vessels.

IV.

The facts already related led me to conclude that alcohol, the essential oil of almonds, the juice of aconite, the oil of tobacco, and the woorara, occasion death simply by destroying the functions of the brain. The following experiment appears fully to establish the truth of this conclusion.

Exp. 30. The temperature of the room being 58° of FAHRENHEIT's thermometer, I made two wounds in the side of a rabbit, and applied to them some of the woorara in the form

of paste. In seven minutes after the application, the hind legs were paralysed, and in fifteen minutes respiration had ceased, and he was apparently dead. Two minutes afterwards the heart was still beating, and a tube was introduced through an opening into the trachea, by means of which the lungs were inflated. The artificial respiration was made regularly about thirty-six times in a minute.

At first, the heart contracted one hundred times in a minute.

At the end of forty minutes, the pulse had risen to one hundred and twenty in a minute.

At the end of an hour, it had risen to one hundred and forty in a minute.

At the end of an hour and twenty-three minutes, the pulse had fallen to a hundred, and the artificial respiration was discontinued.

At the commencement of the experiment, the ball of a thermometer being placed in the rectum, the quicksilver rose to one hundred degrees; at the close of the experiment it had fallen to eighty-eight and a half.

During the continuance of the artificial respiration, the blood in the femoral artery was of a florid red, and that in the femoral vein of a dark colour, as usual.

It has been observed by M. Bichat, that the immediate cause of death, when it takes place suddenly, must be the cessation of the functions of the heart, the brain, or the lungs. This observation may be extended to death under all circumstances. The stomach, the liver, the kidneys, and many other organs are necessary to life, but their constant action is not necessary; and the cessation of their functions cannot therefore be the *immediate* cause of death. As in this case the action

of the heart had never ceased; as the circulation of the blood was kept up by artificial respiration for more than an hour and twenty minutes after the poison had produced its full effects; and as during this time the usual changes in the colour of the blood took place in the lungs; it is evident that the functions of the heart and lungs were unimpaired: but that those of the brain had ceased, is proved, by the animal having continued in a state of complete insensibility, and by this circumstance, that animal heat, to the generation of which I have formerly shewn the influence of the brain to be necessary, was not generated.

Having learned that the circulation might be kept up by artificial respiration for a considerable time after the woorara had produced its full effects, it occurred to me that in an animal under the influence of this or of any other poison that acts in a similar manner, by continuing the artificial respiration for a sufficient length of time after natural respiration had ceased, the brain might recover from the impression, which the poison had produced, and the animal might be restored to life. In the last experiment, the animal gave no sign of returning sensibility; but it is to be observed, 1. That the quantity of the poison employed was very large. 2. That there was a great loss of animal heat, in consequence of the temperature of the room being much below the natural temperature of the animal, which could not therefore be considered under such favourable circumstances as to recovery, as if it had been kept in a higher temperature. 3. That the circulation was still vigorous when I left off inflating the lungs, and therefore it cannot be known what would have been the result, if the artificial respiration had been longer continued.

Exp. 30. A wound was made in the side of a rabbit, and one drop of the essential oil of almonds was inserted into it, and immediately the animal was placed in a temperature of 99°. In two minutes he was under the influence of the poison. The usual symptoms took place, and in three minutes more respiration had ceased, and he lay apparently dead, but the heart was still felt beating through the ribs. A tube was then introduced into one of the nostrils, and the lungs were inflated about thirty-five times in a minute. Six minutes after the commencement of artificial respiration, he moved his head and legs, and made an effort to breathe. He then was seized with convulsions, and again lay motionless, but continued to make occasional efforts to breathe. Sixteen minutes after its commencement, the artificial respiration was discontinued. He now breathed spontaneously seventy times in a minute, and moved his head and extremities. After this, he occasionally rose, and attempted to walk. In the intervals, he continued in a dozing state; but from this he gradually recovered. In less than two hours he appeared perfectly well, and he continued well on the following day.

The inflating the lungs has been frequently recommended in cases of suffocation, where the cause of death is the cessation of the functions of the lungs: as far as I know, it has not been before proposed in those cases, in which the cause of death is the cessation of the functions of the brain.* It is probable that this method of treatment might be employed with advantage for the recovery of persons labouring under the effects of opium, and many other poisons.

* Since this paper was read, I have been favoured by the Right Hon. the President with the perusal of a Dissertation on the Effects of the Upas Tieute, lately published

V.

The experiments, which have been detailed, lead to the following conclusions.

1. Alcohol, the essential oil of almonds, the juice of aconite, the empyreumatic oil of tobacco, and the woorara, act as poisons by simply destroying the functions of the brain; universal death taking place, because respiration is under the influence of the brain, and ceases when its functions are destroyed.

2. The infusion of tobacco when injected into the intestine, and the upas antiar when applied to a wound, have the power of rendering the heart insensible to the stimulus of the blood, thus stopping the circulation; in other words, they occasion syncope.

3. There is reason to believe that the poisons, which in these experiments were applied internally, produce their effects through the medium of the nerves without being absorbed into the circulation.

4. When the woorara is applied to a wound, it produces its effects on the brain, by entering the circulation through the divided blood-vessels, and, from analogy, we may conclude that other poisons, when applied to wounds, operate in a similar manner.

5. When an animal is apparently dead from the influence of a poison, which acts by simply destroying the functions of

at Paris by M. DELILE, by which I find that he had employed artificial respiration for the purpose of recovering animals, which were under the influence of this poison, with success. M. DELILE describes the Upas Tieuté as causing death, by occasioning repeated and long continued contractions of the muscles of respiration, on which it acts through the medium of the spinal marrow, without destroying the functions of the brain.

the brain, it may, in some instances, at least, be made to recover, if respiration is artificially produced, and continued for a certain length of time.

From analogy we might draw some conclusions respecting the mode in which some other vegetable poisons produce their effects on the animal system; but I forbear to enter into any speculative inquiries; as it is my wish, in the present communication, to record such facts only, as appear to be established by actual experiment.

Addition to the Croonian Lecture for the Year 1810.

In the experiments formerly detailed, where the circulation was maintained by means of artificial respiration after the head was removed, I observed that the blood, in its passage through the lungs, was altered from a dark to a scarlet colour, and hence I was led to conclude that the action of the air produced in it changes analogous to those, which occur under ordinary circumstances. I have lately, with the assistance of my friend Mr. W. Brande, made the following experiment, which appears to confirm the truth of this conclusion.

An elastic gum bottle, having a tube and stop-cock connected with it, was filled with about a pint of oxygen gas. The spinal marrow was divided in the neck of a young rabbit, and the blood-vessels having been secured, the head was removed, and the circulation was maintained by inflating the lungs with atmospheric air for five minutes, at the end of which time the tube of the gum bottle was inserted into the trachea, and carefully secured by a ligature, so that no air might escape. By making pressure on the gum bottle, the

gas was made to pass and repass into and from the lungs about thirty times in a minute. At first, the heart acted one hundred and twenty times in a minute, with regularity and strength; the thermometer, in the rectum, rose to 100°. At the end of an hour, the heart acted as frequently as before, but more feebly; the blood in the arteries was very little more florid than that in the veins; the thermometer in the rectum had fallen to 98°. The gum bottle was then removed. On causing a stream of the gas, which it contained, to pass through lime-water, the presence of carbonic acid was indicated by the liquid being instantly rendered turbid. The proportion of carbonic acid was not accurately determined; but it appeared to form about one-half of the quantity of gas in the bottle.

B. C. BRODIE.

ERRATA.

Page 30. line 13. *for* artery *read* ureter.
47, last line of table 4th col. *for* 95 *read* 91½.

From the Press of
W. BULMER & Co.
Cleveland-Row, St. James's,
London.

METEOROLOGICAL JOURNAL,

KEPT AT THE APARTMENTS

OF THE

ROYAL SOCIETY,

BY ORDER OF THE

PRESIDENT AND COUNCIL.

METEOROLOGICAL JOURNAL
for January, 1810.

1810	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds. Points.	Str.	Weather.
	°			°	°	Inches.	°			
Jan. 1	46	8	0	48	52	30,10	75	WJ	2	Cloudy.
	51	2	0	51	56	30,14	72	SW	2	Cloudy.
2	46	8	0	46	52	30,22	75	S	2	Cloudy.
	47	2	0	47	56	30,20	72	S	2	Cloudy.
3	44	8	0	46	53	30,15	73	WSW	1	Cloudy.
	50	2	0	50	56	30,18	73	S	1	Cloudy.
4	45	8	0	45	55	30,25	74	S	1	Cloudy.
	50	2	0	50	57	30,30	73	S	1	Cloudy.
5	47	8	0	47	55	30,34	70	S	1	Cloudy.
	47	2	0	44	57	30,31	68	E	1	Cloudy.
6	42	8	0	42	54	30,30	70	SSW	1	Cloudy.
	49	2	0	49	57	30,31	70	SSW	1	Cloudy.
7	43	8	0	43	54	30,24	70	S	1	Cloudy.
	45	2	0	45	55	30,16	68	S	1	Cloudy.
8	39	8	0	39	53	30,03	68	E	1	Cloudy.
	45	2	0	41	54	29,93	70	ESE	1	Cloudy.
9	41	8	0	46	53	29,84	75	ESE	1	Rain.
	48	2	0	48	55	29,80	73	SE	2	Cloudy.
10	40	8	0	41	52	29,97	70	E	1	Cloudy.
	45	2	0	44	55	29,98	69	ESE	1	Cloudy.
11	41	8	0	44	53	29,91	73	E	1	Cloudy.
	47	2	0	46	55	29,88	68	SE	1	Cloudy.
12	44	8	0	44	53	29,87	70	E	1	Cloudy.
	45	2	0	44	55	29,87	68	E	1	Cloudy.
13	34	8	0	34	51	29,93	65	E	2	Cloudy.
	34	2	0	33	54	29,96	62	E	2	Cloudy.
14	27	8	0	27	48	30,04	60	E	2	Cloudy.
	30	2	0	29	52	29,98	60	E	2	Fair.
15	25	8	0	25	47	29,86	67	NE	2	Snow.
	29	2	0	28	50	29,82	62	NE	2	Fair.
16	18	8	0	20	45	29,75	68	SW	1	Cloudy.
	26	2	0	26	48	29,73	68	SSW	1	Cloudy.

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for January, 1810.

1810	Six'h Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.		Winds. Points.	Str.	Weather.
	° /			°	°	Inches.					
Jan. 17	14	8	0	18	43	30,00	68		WSW	1	Cloudy.
	31	2	0	31	46	30,04	68		SW	1	Fair.
18	25	8	0	28	43	30,23	73		W	1	Foggy.
	37	2	0	37	47	30,24	71		NE	1	Snow.
19	23	8	0	23	43	30,24	73		NE	1	Foggy.
	33	2	0	33	47	30,22	70		ENE	1	Foggy.
20	18	8	0	20	42	30,17	72		NE	1	Foggy.
	26	2	0	26	46	30,13	70		NE	1	Foggy.
21	24	8	0	28	42	30,02	74		NE	1	Cloudy.
	36	2	0	36	46	29,99	70		NE	1	Cloudy.
22	29	8	0	30	42	29,94	72		NE	1	Cloudy.
	35	2	0	35	45	29,91	68		NE	1	Cloudy.
23	33	8	0	34	43	30,08	75		NE	1	Foggy.
	38	2	0	38	46	30,13	75		NNE	1	Cloudy.
24	35	8	0	35	43	30,18	75		NE	1	Cloudy.
	38	2	0	38	47	30,20	73		NE	1	Cloudy.
25	33	8	0	33	43	30,27	72		ESE	1	Cloudy.
	35	2	0	35	47	30,30	72		SSW	1	Cloudy.
26	31	8	0	31	45	30,33	73		NE	1	Cloudy.
	37	2	0	37	47	30,28	73		NE	1	Cloudy.
27	32	8	0	32	46	30,26	73		E	1	Cloudy.
	32	2	0	32	48	30,23	73		SE	1	Cloudy.
28	30	8	0	30	44	30,22	72		N	1	Cloudy.
	31	2	0	31	47	30,21	72		N	1	Cloudy.
29	30	8	0	31	44	30,28	72		N	1	Cloudy.
	34	2	0	34	46	30,32	72		E	1	Cloudy.
30	31	8	0	32	44	30,44	72		E	1	Cloudy.
	33	2	0	33	46	30,44	72		E	1	Cloudy.
31	30	8	0	31	43	30,36	74		SE	1	Foggy.
	45	2	0	43	47	30,25	75		S	1	Cloudy.

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for February, 1810.

1810	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.		Winds.		Weather.
		H.	M.			Inches.			Points.	Str.	
	°			°	°		°				
Feb. 1	43	7	0	46	46	30,16	78		SSW	1	Cloudy.
	50	2	0	50	50	30,11	78		SW	1	Cloudy.
2	46	7	0	46	49	30,07	78		SSW	1	Cloudy.
	47	2	0	47	52	30,02	78		SW	1	Rain.
3	43	7	0	43	50	29,79	78		S	1	Foggy.
	47	2	0	46	53	29,68	78		S	1	Cloudy.
4	43	7	0	43	51	29,88	76		N	1	Cloudy.
	45	2	0	45	53	30,00	67		NE	1	Cloudy.
5	35	7	0	35	50	30,08	72		N	1	Fair.
	46	2	0	46	53	30,04	71		S	1	Cloudy.
6	39	7	0	39	50	30,03	74		SSW	1	Fair.
	47	2	0	47	53	30,00	70		SW	1	Cloudy.
7	45	7	0	45	52	29,94	77		SSW	1	Cloudy.
	49	2	0	49	53	29,92	74		SSW	1	Cloudy.
8	45	7	0	45	52	29,95	76		S	1	Cloudy.
	48	2	0	48	54	29,95	74		S	2	Cloudy.
9	43	7	0	43	52	29,79	73		S	2	Cloudy.
	48	2	0	47	54	29,73	74		S	2	Cloudy.
10	47	7	0	47	53	29,76	76		SSW	1	Cloudy.
	51	2	0	51	56	29,80	66		N	1	Cloudy.
11	43	7	0	43	54	29,87	69		E	1	Cloudy.
	43	2	0	43	56	29,78	69		E	2	Cloudy.
12	40	7	0	40	53	29,41	76		E	1	Rain.
	42	2	0	42	54	29,31	76		E	1	Rain.
13	38	7	0	38	53	29,02	72		W	1	Cloudy.
	45	2	0	45	55	29,01	60		W	2	Fair.
14	36	7	0	36	52	29,32	70		NW	1	Cloudy.
	40	2	0	40	53	29,38	66		N	1	Cloudy.
15	36	7	0	36	52	29,69	63		NE	2	Cloudy.
	41	2	0	41	53	29,80	61		NE	2	Cloudy.
16	28	7	0	28	51	30,08	64		N	1	Cloudy.
	38	2	0	38	53	30,05	60		NW	1	Fair.

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for February, 1810.

1810	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds. Points.	Winds. Str.	Weather.
	°					Inches.				
Feb. 17	27	7	0	27	50	30,12	70	NNE	1	Cloudy.
	36	2	0	36	53	30,17	66	NNE	1	Fair.
18	24	7	0	24	48	30,18	65	NW	1	Cloudy.
	35	2	0	35	50	30,09	63	SW	1	Snow.
19	29	7	0	29	47	29,98	65	N	1	Cloudy.
	35	2	0	35	50	30,04	60	N	2	Fine.
20	27	7	0	27	46	30,20	69	NNE	1	Snow.
	32	2	0	32	49	30,28	61	NE	1	Fair.
21	18	7	0	19	43	30,44	66	NE	1	Cloudy.
	31	2	0	30	47	30,44	62	W	1	Cloudy.
22	25	7	0	26	43	30,28	63	W	1	Cloudy.
	36	2	0	36	46	30,18	60	W	2	Cloudy.
23	30	7	0	34	44	29,54	73	SW	2	Rain.
	46	2	0	45	47	29,39	75	SW	2	Cloudy.
24	40	7	0	41	46	29,54	76	W	1	Cloudy.
	51	2	0	50	49	29,55	75	WSW	2	Cloudy.
25	47	7	0	48	49	29,47	67	WSW	1	Cloudy.
	51	2	0	49	53	29,50	58	NW	2	Cloudy.
26	37	7	0	38	48	29,87	63	W	1	Cloudy.
	47	2	0	47	52	29,93	57	W	1	Fair.
27	45	7	0	48	51	29,72	75	WSW	1	Cloudy.
	54	2	0	54	53	29,66	68	SW	2	Cloudy.
28	42	7	0	43	52	30,01	65	SW	1	Cloudy.
	54	2	0	53	56	30,03	61	SW	1	Fair.

METEOROLOGICAL JOURNAL
for March, 1810.

1810	Six's Therm. least and greatest Heat.	Time.	Therm. without.	Therm. within.	Barom.	Hygrometer.		Winds.		Weather.
		H. M.	°	°	Inches.			Points.	Str.	
	°					°				
Mar. 1	48	7 0	48	53	29,89	75		SW	2	Cloudy.
	55	2 0	54	56	29,80	67		SW	2	Cloudy.
2	49	7 0	50	55	29,70	76		SW	1	Cloudy.
	54	2 0	53	57	29,70	73		SW	1	Cloudy.
3	48	7 0	49	56	29,61	76		SSW	1	Cloudy.
	54	2 0	53	58	29,56	74		S	1	Rain.
4	40	7 0	40	55	29,70	76		SW	1	Cloudy.
	50	2 0	49	56	29,61	70		S	1	Cloudy.
5	42	7 0	42	55	29,28	72		WNW	1	Cloudy.
	47	2 0	47	56	29,17	63		N	1	Fair.
6	36	7 0	36	54	28,93	75		NE	1	Snow.
	43	2 0	40	56	28,95	72		NE	1	Cloudy.
7	40	7 0	42	54	28,92	73		E	1	Cloudy.
	45	2 0	45	56	28,84	73		E	2	Rain.
8	40	7 0	44	53	28,95	73		E	2	Cloudy.
	52	2 0	51	56	29,00	67		S	2	Cloudy.
9	47	7 0	50	56	29,12	75		S	2	Cloudy.
	58	2 0	56	57	29,25	68		S	2	Rain.
10	50	7 0	52	56	29,42	65		SW	1	Cloudy.
	59	2 0	58	59	29,58	57		W	2	Fair.
11	47	7 0	49	57	29,92	73		SW	1	Cloudy.
	58	2 0	57	60	29,94	63		SW	1	Cloudy.
12	51	7 0	52	58	29,70	70		W	1	Cloudy.
	57	2 0	56	61	29,68	65		WSW	1	Cloudy.
13	41	7 0	41	57	30,01	64		E	1	Cloudy.
	45	2 0	45	58	30,04	63		E	1	Cloudy.
14	40	7 0	40	56	30,02	61		NE	1	Cloudy.
	44	2 0	43	58	30,02	58		NE	1	Cloudy.
15	35	7 0	36	54	29,98	59		E	2	Cloudy.
	41	2 0	40	56	29,73	56		E	2	Cloudy.
16	33	7 0	34	53	29,55	62		NE	2	Cloudy.
	41	2 0	40	56	29,60	60		N	1	Cloudy.

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1810	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds. Points.	Str.	Weather.
	°			°	°	Inches.	°			
Mar. 17	34	7	0	36	53	29,67	62	NNW	2	Fair.
	43	2	0	42	56	29,70	53	N	2	Fair.
18	30	7	0	34	52	29,89	60	NNE	2	Fair.
	40	2	0	40	56	29,91	55	NE	2	Fair.
19	28	7	0	30	51	30,01	03	W	1	Fair.
	46	2	0	44	56	30,05	55	W	1	Fair.
20	34	7	0	36	52	30,05	66	NW	1	Hazy.
	50	2	0	49	55	30,00	60	NNE	1	Hazy.
21	33	7	0	38	50	29,76	65	WNW	1	Cloudy.
	50	2	0	49	55	29,74	49	NNW	1	Cloudy.
22	33	7	0	37	50	29,93	65	N	1	Fair.
	44	2	0	43	56	30,03	49	NNE	1	Hazy.
23	30	7	0	36	52	29,98	61	NE	1	Hazy.
	49	2	0	49	57	29,86	56	W	1	Hazy.
24	30	7	0	35	52	29,76	64	E	1	Fair.
	47	2	0	46	56	29,76	58	E	2	Hazy.
25	36	7	0	37	51	29,78	60	E	2	Hazy.
	42	2	0	41	54	29,81	56	E	2	Cloudy.
26	34	7	0	36	50	29,93	60	E	2	Hazy.
	43	2	0	42	54	29,92	57	E	2	Hazy.
27	38	7	0	43	51	29,77	66	E	1	Cloudy.
	56	2	0	51	55	29,66	67	ESE	1	Cloudy.
28	43	7	0	44	52	29,65	67	NW	2	Fair.
	52	2	0	52	55	29,73	68	WNW	2	Fair.
29	39	7	0	43	52	29,91	68	WNW	1	Cloudy.
	54	2	0	53	56	29,92	55	WNW	1	Cloudy.
30	39	7	0	40	54	29,94	65	WNW	1	Cloudy.
	51	2	0	50	55	29,88	57	SW	1	Cloudy.
31	44	7	0	46	54	29,77	69	S	1	Cloudy.
	52	2	0	52	56	29,68	62	SSE	1	Cloudy.

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for April, 1810.

1810	Six's Therm. least and greatest Heat.	Time.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds.		Weather.
		H. M.	°	°	Inches.	°	Points.	Str.	
Apr. 1	43	7 0	44	53	29.43	70	ESE	1	Rain.
	51	2 0	50	55	29.43	64	E	2	Cloudy.
2	44	7 0	44	53	29.70	71	E	1	Cloudy.
	48	2 0	48	55	29.76	65	E	1	Cloudy.
3	44	7 0	46	54	29.63	73	S	1	Cloudy.
	57	2 0	56	58	29.56	69	S	2	Cloudy.
4	43	7 0	44	54	29.43	70	W	2	Rain.
	50	2 0	49	55	29.57	63	WNW	2	Cloudy.
5	37	7 0	41	53	29.76	67	W	2	Cloudy.
	52	2 0	50	57	29.69	53	WSW	2	Cloudy.
6	44	7 0	44	54	29.36	65	E	2	Cloudy.
	48	2 0	47	55	29.29	63	E	2	Cloudy.
7	43	7 0	46	54	29.32	68	E	2	Cloudy.
	54	2 0	54	57	29.36	60	SE	2	Cloudy.
8	42	7 0	45	54	29.48	70	E	2	Cloudy.
	54	2 0	53	56	29.48	57	E	2	Cloudy.
9	44	7 0	47	53	29.46	74	E	2	Cloudy.
	54	2 0	53	56	29.44	63	E	2	Cloudy.
10	44	7 0	44	54	29.52	75	NE	1	Rain.
	45	2 0	45	56	29.57	70	N	2	Rain.
11	30	7 0	38	52	29.66	68	NNE	2	Cloudy.
	45	2 0	43	54	29.68	65	N	2	Cloudy.
12	34	7 0	36	51	29.91	64	NE	2	Cloudy.
	43	2 0	43	53	29.93	54	NNE	2	Cloudy.
13	31	7 0	35	51	29.94	64	N	2	Cloudy.
	44	2 0	43	53	29.93	55	NNE	2	Cloudy.
14	35	7 0	38	51	29.90	65	N	2	Cloudy.
	46	2 0	45	52	29.88	54	WNW	1	Cloudy.
15	40	7 0	44	51	29.82	60	SSW	1	Cloudy.
	52	2 0	50	53	29.79	56	SSW	2	Fair.
16	38	7 0	42	51	29.58	62	E	2	Hazy.
	51	2 0	49	51	29.45	54	E	2	Cloudy.

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1810	Six't Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.		Winds. Points.	Str.	Weather.
	°			°	°	Inches.	°				
Apr. 17	39	7	0	42	50	29,46	68		W	2	Fair.
	54	2	0	52	54	29,54	59		SSW	2	Fair.
18	39	7	0	47	52	29,63	71		SSE	2	Hazy.
	59	2	0	57	56	29,62	54		SE	2	Fair.
19	46	7	0	49	53	29,74	68		SSW	2	Fair.
	60	2	0	59	56	29,78	66		ESE	2	Fair.
20	45	7	0	48	54	29,93	65		SW	2	Cloudy.
	61	2	0	60	56	30,07	60		SW	2	Cloudy.
21	46	7	0	49	54	30,17	67		W	1	Fair.
	62	2	0	62	58	30,19	55		W	2	Fair.
22	44	7	0	48	55	30,20	67		W	1	Cloudy.
	63	2	0	63	57	30,18	54		WNW	2	Fine.
23	48	7	0	53	56	30,18	65		W	1	Hazy.
	66	2	0	66	59	30,19	57		E	1	Fair.
24	47	7	0	48	57	30,21	67		E	1	Hazy.
	63	2	0	63	59	30,19	58		E	1	Fair.
25	44	7	0	49	57	30,17	62		NE	1	Hazy.
	58	2	0	58	59	30,14	63		ENE	2	Fair.
26	44	7	0	49	56	30,12	63		E	1	Hazy.
	57	2	0	57	58	30,12	60		E	1	Fair.
27	46	7	0	50	56	30,10	59		E	1	Hazy.
	57	2	0	57	59	30,09	49		E	1	Fine.
28	45	7	0	49	56	30,09	60		E	1	Hazy.
	65	2	0	64	60	30,10	49		E	1	Fine.
29	44	7	0	49	58	30,10	59		NNE	1	Hazy.
	69	2	0	69	62	30,04	46		E	1	Fine.
30	48	7	0	53	59	29,99	55		NNE	1	Hazy.
	69	2	0	69	61	29,94	49		E	2	Fine.

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for May, 1810.

1810		Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.		Winds. Points.	Winds. Str.	Weather.
		°			°	°	Inches.					
May	1	48	7	0	51	59	29,86	56		NNE	1	Hazy.
		65	2	0	64	61	29,84	51		NE	1	Fine.
	2	44	7	0	46	59	29,87	56		NNE	1	Hazy.
		56	2	0	55	59	29,84	54		NNE	1	Cloudy.
	3	44	7	0	46	58	29,75	63		NW	1	Rain.
		52	2	0	52	59	29,73	60		N	2	Cloudy.
	4	40	7	0	45	57	29,72	63		W	1	Cloudy.
		55	2	0	55	57	29,75	57		NNE	1	Cloudy.
	5	39	7	0	43	55	29,85	56		NE	1	Cloudy.
		50	2	0	49	55	29,84	52		E	1	Cloudy.
	6	34	7	0	40	54	29,81	58		NNE	1	Fair.
		51	2	0	50	55	29,80	49		N	1	Cloudy.
	7	40	7	0	44	53	29,70	57		NE	2	Cloudy.
		52	2	0	48	54	29,56	69		ENE	1	Cloudy.
	8	48	7	0	52	54	29,41	75		SW	1	Cloudy.
		63	2	0	62	57	29,55	48		WNW	1	Cloudy.
	9	45	7	0	50	55	29,77	65		SW	1	Hazy.
		62	2	0	62	57	29,78	53		SW	2	Cloudy.
	10	48	7	0	52	56	29,94	59		WSW	1	Cloudy.
		65	2	0	62	60	29,96	57		N	1	Cloudy.
	11	47	7	0	49	57	30,19	59		ENE	1	Cloudy.
		57	2	0	56	59	30,07	54		E	1	Fair.
	12	43	7	0	46	57	30,00	60		E	1	Hazy.
		60	2	0	59	59	29,96	57		E	2	Cloudy.
	13	49	7	0	51	57	29,80	64		E	2	Hazy.
		53	2	0	52	58	29,82	62		E	2	Cloudy.
	14	50	7	0	52	57	29,67	61		E	2	Hazy.
		60	2	0	59	59	29,60	55		E	2	Fair.
	15	47	7	0	50	57	29,39	73		N	1	Rain.
		57	2	0	56	58	29,35	68		WNW	1	Cloudy.
	16	48	7	0	49	56	29,39	68		WNW	1	Rain.
		63	2	0	62	58	29,43	54		WSW	1	Cloudy.

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1810	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.	°			
May 17	47	7	0	50	57	29,61	56	N	1	Cloudy.
	59	2	0	59	56	29,50	64	SE	1	Rain.
18	43	7	0	43	56	29,55	71	N	1	Rain.
	50	2	0	50	56	29,63	65	N	1	Rain.
19	41	7	0	43	55	29,92	66	W	1	Hazy.
	60	2	0	59	57	29,97	52	W	1	Cloudy.
20	41	7	0	50	55	29,99	62	E	1	Cloudy.
	61	2	0	60	58	29,87	53	E	1	Cloudy.
21	48	7	0	52	57	29,65	62	SW	2	Cloudy.
	63	2	0	62	59	29,62	54	SW	2	Cloudy.
22	48	7	0	52	57	29,84	64	W	1	Cloudy.
	64	2	0	62	58	29,89	54	W	1	Cloudy.
23	48	7	0	51	57	30,09	56	N	1	Cloudy.
	62	2	0	60	59	30,10	49	NNE	1	Cloudy.
24	45	7	0	50	57	30,18	58	W	1	Hazy.
	64	2	0	63	60	30,14	48	W	1	Cloudy.
25	49	7	0	51	58	30,06	62	N	1	Cloudy.
	64	2	0	64	59	30,08	57	N	1	Cloudy.
26	44	7	0	48	57	30,04	65	E	1	Hazy.
	65	2	0	63	60	29,97	49	E	1	Cloudy.
27	46	7	0	52	59	29,93	59	E	1	Cloudy.
	67	2	0	67	60	29,90	50	E	2	Cloudy.
28	48	7	0	52	59	30,04	57	NE	2	Cloudy.
	60	2	0	60	60	30,23	50	NE	2	Cloudy.
29	43	7	0	49	58	30,38	57	ENE	1	Hazy.
	60	2	0	59	60	30,38	50	E	2	Fine.
30	43	7	0	50	58	30,36	61	NNE	1	Hazy.
	64	2	0	64	60	30,34	50	ESE	2	Fine.
31	47	7	0	52	58	30,32	56	E	1	Hazy.
	66	2	0	66	61	30,31	51	E	1	Fine.

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for June, 1810.

1810	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds. Points.	Str.	Weather.
	o			o	o	Inches.				
June 1	49	7	o	55	59	30,28	61	E	1	Hazy.
	66	2	o	66	61	30,29	48	E	2	Fine.
2	47	7	o	52	59	30,24	61	E	1	Hazy.
	69	2	o	69	62	30,22	51	E	2	Fair.
3	49	7	o	54	60	30,22	59	NNE	1	Fair.
	67	2	o	67	61	30,23	53	NE	2	Cloudy.
4	46	7	o	50	59	30,26	61	ENE	1	Fair.
	68	2	o	68	62	30,19	48	E	1	Fine.
5	53	7	o	55	60	30,23	60	NNE	2	Cloudy.
	60	2	o	60	59	30,25	56	NNE	2	Cloudy.
6	49	7	o	53	59	30,23	57	N	1	Fair.
	71	2	o	71	61	30,17	52	ENE	2	Fair.
7	55	7	o	58	61	30,14	63	E	1	Cloudy.
	72	2	o	72	62	30,11	50	N	1	Fair.
8	54	7	o	60	61	30,09	61	WSW	1	Cloudy.
	74	2	o	74	63	30,04	53	SW	1	Fair.
9	58	7	o	59	62	29,96	59	N	1	Cloudy.
	73	2	o	73	64	29,92	54	E	1	Fair.
10	58	7	o	62	62	29,73	63	E	1	Cloudy.
	72	2	o	72	64	29,68	58	NNE	1	Cloudy.
11	51	7	o	56	62	29,80	58	NW	1	Fair.
	70	2	o	70	64	29,83	50	NW	2	Cloudy.
12	54	7	o	60	62	29,94	55	WNW	2	Cloudy.
	66	2	o	66	62	29,94	51	WNW	2	Cloudy.
13	53	7	o	60	61	29,89	57	SW	2	Cloudy.
	69	2	o	69	62	29,91	52	SW	2	Rain.
14	46	7	o	52	60	30,08	59	NW	2	Fair.
	65	2	o	65	61	30,15	49	NW	2	Fair.
15	47	7	o	53	60	30,24	58	WNW	1	Cloudy.
	69	2	o	69	61	30,14	53	NW	2	Fair.
16	48	7	o	53	60	30,08	60	N	2	Cloudy.
	65	2	o	65	62	29,99	51	NE	2	Cloudy.

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1810	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds. Points.	Str.	Weather.
	°			°	°	Inches.	°			
June 17	45	7	0	53	60	29.97	56	NE	2	Cloudy.
	60	2	0	66	61	29,92	52	NW	2	Fair.
18	50	7	0	56	60	29,93	55	WNW	1	Fair.
	75	2	0	75	62	29,93	48	WNW	1	Fine.
19	58	7	0	63	62	29,96	60	WNW	2	Cloudy.
	70	2	0	70	63	29,95	56	WSW	1	Cloudy.
20	59	7	0	62	63	29,94	64	WNW	2	Cloudy.
	73	2	0	73	64	29,96	60	WNW	2	Cloudy.
21	56	7	0	60	62	30,16	60	WNW	1	Fair.
	78	2	0	78	66	30,18	48	NW	1	Fair.
22	62	7	0	65	65	30,28	61	N	1	Cloudy.
	75	2	0	75	66	30,29	56	ENE	1	Fair.
23	53	7	0	60	60	30,38	63	E	1	Fair.
	70	2	0	70	65	30,32	58	ENE	1	Fair.
24	50	7	0	58	60	30,24	63	E	1	Cloudy.
	72	2	0	72	65	30,17	55	E	1	Fine.
25	55	7	0	59	62	30,13	63	E	1	Cloudy.
	77	2	0	77	66	30,10	61	E	1	Fine.
26	59	7	0	60	63	30,06	59	NNE	2	Cloudy.
	64	2	0	62	63	30,05	59	NNE	1	Cloudy.
27	53	7	0	56	63	30,01	58	NNW	1	Fair.
	68	2	0	68	63	29,94	52	N	1	Cloudy.
28	52	7	0	57	59	29,90	61	E	1	Cloudy.
	71	2	0	71	64	29,98	55	E	1	Fair.
29	57	7	0	61	62	29,96	64	E	1	Cloudy.
	74	2	0	74	65	29,98	56	E	1	Cloudy.
30	54	7	0	59	63	30,10	64	W	1	Fair.
	71	2	0	71	66	30,11	50	WSW	1	Fair.

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1810	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds. Points.	Str.	Weather.
	°			°	°	Inches.	°			
July 1	58	7	0	60	64	30,08	57	SE	2	Fair.
	75	2	0	75	67	30,06	55	SE	2	Fair.
2	58	7	0	61	64	29,82	57	W	2	Cloudy.
	75	2	0	75	66	29,81	50	WSW	2	Cloudy.
3	58	7	0	62	65	29,66	61	SSE	2	Cloudy.
	67	2	0	67	65	29,47	63	SE	1	Rain.
4	55	7	0	60	63	29,38	61	W	2	Cloudy.
	65	2	0	63	64	29,38	62	W	2	Cloudy.
5	54	7	0	59	64	29,75	64	WSW	1	Cloudy.
	70	2	0	70	64	29,78	57	SW	1	Cloudy.
6	60	7	0	63	64	29,89	62	W	1	Cloudy.
	73	2	0	73	65	29,94	51	W	1	Cloudy.
7	56	7	0	64	62	30,01	57	SW	1	Fair.
	76	2	0	76	65	29,99	52	SW	1	Fair.
8	59	7	0	66	66	29,78	60	S	2	Cloudy.
	71	2	0	71	66	29,76	65	S	1	Cloudy.
9	55	7	0	61	64	29,93	61	S	2	Cloudy.
	72	2	0	72	66	29,89	52	S	2	Cloudy.
10	60	7	0	64	64	29,74	64	S	2	Cloudy.
	73	2	0	70	65	29,74	57	S	2	Fair.
11	61	7	0	64	65	29,62	64	S	2	Cloudy.
	78	2	0	77	68	29,53	54	ESE	2	Cloudy.
12	61	7	0	63	66	29,53	63	SW	2	Fair.
	73	2	0	72	66	29,58	56	SW	1	Cloudy.
13	60	7	0	65	65	29,59	60	WSW	2	Cloudy.
	71	2	0	69	66	29,57	58	WSW	2	Fair.
14	57	7	0	62	65	29,69	60	W	2	Fair.
	71	2	0	61	66	29,76	63	W	2	Rain. [Thunder and lightning.
15	52	7	0	59	64	29,92	62	NW	1	Cloudy.
	71	2	0	68	65	29,96	54	W	1	Fine.
16	51	7	0	58	64	30,08	63	W	1	Cloudy.
	70	2	0	65	65	30,03	54	W	1	Cloudy.

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1810	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds. Points.	Str.	Weather.
	°			°	°	Inches.				
July 17	53	7	0	57	63	29.80	63	N	1	Cloudy.
	68	2	0	68	63	29,77	54	W	1	Cloudy.
18	54	7	0	57	62	29,55	66	E	2	Cloudy.
	67	2	0	65	63	29,60	54	N	2	Cloudy.
19	56	7	0	58	63	29,83	62	NNW	1	Cloudy.
	69	2	0	68	64	29,83	52	W	1	Cloudy.
20	53	7	0	57	63	29.78	62	NNW	1	Fair.
	68	2	0	66	64	29.78	54	N	1	Fair.
21	54	7	0	57	63	29.92	58	NW	1	Fair.
	62	2	0	62	64	30,00	55	NW	1	Fair.
22	52	7	0	56	62	30,09	57	NNW	1	Fair.
	66	2	0	65	63	30,11	52	WNW	1	Cloudy.
23	52	7	0	56	62	30,16	61	SSW	1	Fair.
	73	2	0	71	64	30,16	50	W	1	Fair.
24	55	7	0	59	63	30,14	63	WSW	1	Fair.
	73	2	0	73	65	30,07	52	S	1	Fine.
25	54	7	0	60	63	29.84	59	S	1	Fair.
	70	2	0	72	67	29.78	55	SE	2	Cloudy.
26	61	7	0	65	65	29,78	62	SSW	1	Fair.
	72	2	0	72	65	29,78	58	SW	1	Rain.
27	55	7	0	60	64	29.62	64	S	2	Cloudy.
	67	2	0	67	64	29.3	64	SSE	2	Rain.
28	54	7	0	68	63	29.55	65	WSW	1	Fair.
	70	2	0	62	63	29,61	62	W	1	Cloudy.
29	53	7	0	60	63	29.90	64	W	1	Fair.
	70	2	0	70	64	29,94	56	W	1	Cloudy.
30	58	7	0	61	63	29,71	64	WSW	2	Fair.
	69	2	0	68	64	29,68	58	W	2	Cloudy.
31	54	7	0	60	62	29,81	61	W	2	Fair.
	71	2	0	65	64	29,80	60	SW	2	Cloudy.

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1810	Six's Therm least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom. Inches.	Hygrometer.	Winds. Points.	Str.	Weather.
	°			°	°		°			
Aug. 1	52	7	0	60	60	29,80	60	W	2	Fair.
	68	2	0	64	62	29,73	58	NW	2	Cloudy.
2	53	7	0	57	60	29,94	66	W	2	Fair.
	71	2	0	71	64	29,94	56	W	1	Cloudy.
3	60	7	0	63	62	29,84	67	NW	2	Cloudy.
	71	2	0	71	65	29,79	57	WNW	2	Cloudy.
4	57	7	0	61	62	29,58	67	ESE	2	Cloudy.
	68	2	0	67	63	29,53	58	S	2	Cloudy.
5	55	7	0	60	63	29,72	64	W	2	Fair.
	70	2	0	65	64	29,65	63	SSW	2	Rain. [Thunder.
6	52	7	0	57	62	29,73	64	W	2	Cloudy.
	71	2	0	70	65	29,76	54	WSW	2	Cloudy.
7	52	7	0	60	62	29,65	70	S	2	Rain.
	70	2	0	70	64	29,61	59	WSW	2	Cloudy.
8	60	7	0	62	64	29,54	67	W	1	Rain.
	70	2	0	70	65	29,62	61	W	1	Cloudy.
9	55	7	0	57	63	29,98	64	WNW	2	Cloudy.
	67	2	0	65	63	30,02	54	WNW	2	Cloudy.
10	59	7	0	62	62	29,77	72	W	2	Cloudy.
	71	2	0	71	65	29,77	51	W	2	Fair.
11	55	7	0	58	59	29,59	63	W	2	Fair.
	70	2	0	62	62	29,57	59	WSW	2	Cloudy.
12	59	7	0	63	63	29,52	64	W	2	Cloudy.
	70	2	0	70	65	29,56	54	W	2	Fair.
13	59	7	0	64	63	29,47	65	NW	2	Cloudy.
	68	2	0	67	64	29,69	52	WNW	2	Cloudy.
14	57	7	0	62	63	29,80	64	SW	2	Cloudy.
	67	2	0	64	63	29,78	62	SW	2	Cloudy.
15	58	7	0	61	63	29,05	60	W	1	Cloudy.
	69	2	0	68	63	29,49	54	W	2	Cloudy.
16	53	7	0	53	62	29,48	70	W	2	Rain.
	56	2	0	56	62	29,63	66	N	2	Cloudy.

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1810	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds. Points.	Str.	Weather.
	°			°	°	Inches.				
Aug. 17	50	7	0	52	60	29.84	64	W	1	Cloudy.
	63	2	0	62	62	29.94	54	NW	1	Fair.
18	48	7	0	51	60	30,18	65	W	1	Cloudy.
	69	2	0	67	62	30,20	54	W	1	Fine.
19	54	7	0	58	62	30,17	67	W	1	Cloudy.
	67	2	0	67	62	30,17	67	W	1	Cloudy.
20	51	7	0	52	61	30,28	66	W	1	Fair.
	71	2	0	69	63	30,27	56	W	1	Fine.
21	53	7	0	55	62	30,22	68	W	1	Fair.
	70	2	0	70	65	30,20	56	W	1	Fine.
22	53	7	0	54	63	30,14	68	W	1	Fair.
	74	2	0	74	66	30,05	57	SSW	1	Fine.
23	57	7	0	60	63	29,99	67	N	1	Hazy.
	78	2	0	76	69	29,99	56	S	2	Fine.
24	59	7	0	61	64	30,03	66	ESE	1	Hazy.
	79	2	0	78	67	30,12	55	SE	1	Fine.
25	60	7	0	63	66	29,99	65	ENE	1	Hazy.
	80	2	0	80	71	29,97	56	WSW	2	Fine.
26	61	7	0	63	66	29,95	67	W	1	Hazy.
	73	2	0	73	67	29,95	58	W	2	Cloudy.
27	57	7	0	61	64	30,07	63	N	1	Cloudy.
	74	2	0	73	67	30,07	59	W	2	Cloudy.
28	56	7	0	59	66	30,14	63	W	1	Hazy.
	72	2	0	71	67	30,16	53	N	1	Fine.
29	56	7	0	58	66	30,13	65	SSE	1	Fair.
	72	2	0	70	67	30,10	55	WSW	1	Fair.
30	61	7	0	62	66	30,01	66	W	1	Hazy.
	77	2	0	77	69	29,97	55	WSW	1	Cloudy.
31	61	7	0	62	67	29,88	66	E	1	Cloudy [Th. and L. last night.
	80	2	0	80	70	29,88	57	SSE	1	Fine.

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1810	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.	°			
Sep. 1	66	7	0	66	69	29,86	67	E	1	Fair.
	81	2	0	80	75	29,86	57	SE	1	Fine.
2	67	7	0	68	70	29,91	66	W	1	Fair.
	83	2	0	83	76	29,89	53	E	2	Fine.
3	65	7	0	66	70	29,83	65	W	2	Cloudy.
	75	2	0	72	71	29,82	55	W	2	Cloudy.
4	57	7	0	59	68	29,68	69	W	1	Cloudy.
	65	2	0	62	68	29,74	58	WNW	2	Cloudy.
5	51	7	0	53	66	30,00	65	W	1	Hazy.
	66	2	0	66	68	30,03	54	W	1	Fine.
6	56	7	0	58	66	29,98	66	W	1	Fair.
	69	2	0	66	66	30,03	58	NW	1	Cloudy.
7	49	7	0	52	64	30,33	63	N	1	Fair.
	64	2	0	64	66	30,34	55	E	1	Fine.
8	49	7	0	52	64	30,20	63	N	1	Fair.
	67	2	0	67	66	30,13	55	E	1	Fine.
9	50	7	0	52	61	30,08	65	SE	1	Fair.
	70	2	0	69	65	30,03	57	SW	1	Fine.
10	53	7	0	55	62	29,94	67	W	1	Cloudy.
	70	2	0	70	65	29,92	60	W	1	Cloudy.
11	54	7	0	54	63	29,82	59	WSW	1	Cloudy.
	62	2	0	62	63	29,79	60	S	1	Rain.
12	50	7	0	52	60	29,64	79	NE	1	Rain.
	59	2	0	59	61	29,77	62	SE	1	Cloudy.
13	48	7	0	51	60	30,07	65	NW	1	Cloudy.
	64	2	0	63	62	30,07	52	W	1	Cloudy.
14	57	7	0	59	61	30,12	73	W	1	Cloudy.
	70	2	0	70	64	30,17	57	NE	1	Fair.
15	54	7	0	54	61	30,36	62	BNE	2	Fair.
	62	2	0	62	61	30,36	53	E	1	Fair.
16	55	7	0	55	60	30,27	62	NE	1	Fair.
	65	2	0	64	61	30,21	56	E	1	Cloudy.

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1810	Six's Therm. least and greatest Heat.	Time. H.	Time. M.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds. Points.	Winds. Str.	Weather.
	°			°	°	Inches.	°			
Sep. 17	61	7	0	60	61	30,09	71	NNE	1	Cloudy.
		2	0	69	62	30,07	63	ESE	1	Cloudy.
18		7	0	57	60	30,02	70	NE	2	Fair.
		2	0	67	62	30,02	65	ENE	1	Cloudy.
19		7	0	59	62	30,08	72	N	1	Cloudy.
		2	0	66	63	30,09	63	N	1	Fair.
20		7	0	56	61	30,09	71	E	1	Foggy.
	62	2	0	62	62	30,08	71	E	1	Cloudy.
21	57	7	0	59	62	30,07	72	N	1	Cloudy.
	69	2	0	68	63	30,05	61	E	1	Fair.
22	57	7	0	57	62	30,00	73	SE	1	Foggy.
	70	2	0	70	64	29,92	63	E	1	Cloudy.
23	53	7	0	55	62	29,89	65	S	2	Cloudy.
	61	2	0	61	62	29,92	58	SE	1	Cloudy.
24	54	7	0	55	61	30,05	71	N	1	Cloudy.
	67	2	0	66	62	30,07	55	NNE	1	Cloudy.
25	58	7	0	58	62	30,13	70	N	2	Cloudy.
	71	2	0	71	64	30,11	56	E	1	Fine.
26	55	7	0	57	62	30,08	72	NNE	1	Hazy.
	69	2	0	68	64	30,06	51	E	2	Fine.
27	53	7	0	55	63	29,98	71	N	1	Hazy.
	68	2	0	68	65	29,92	54	E	1	Fine.
28	54	7	0	56	63	29,89	72	N	1	Hazy.
	71	2	0	71	66	29,93	59	NNE	1	Fine.
29	50	7	0	53	62	30,05	68	W	1	Foggy.
	67	2	0	67	65	30,06	61	S	1	Fine.
30	58	7	0	61	63	29,97	71	E	1	Fair.
	70	2	0	70	66	29,98	63	SE	2	Fine.

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1810	Six's Therm. least and greatest Heat.	Time.	Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds.		Weather.
		H. M.			Inches.		Points.	Str.	
	°					°			
Oct. 1	60	7 0	61	61	30,13	69	E	1	Cloudy.
	68	2 0	68	64	30,10	62	E	1	Fine.
2	51	7 0	56	63	30,24	69	NE	2	Fair.
	66	2 0	66	65	30,25	61	ESE	2	Fine.
3	49	7 0	53	62	30,24	68	NE	1	Fine.
	66	2 0	66	64	30,24	55	E	2	Fine.
4	50	7 0	52	62	30,25	71	NE	1	Fine.
	67	2 0	67	64	30,25	60	ESE	2	Fine.
5	50	7 0	51	62	30,16	70	NE	1	Fine.
	65	2 0	65	64	30,07	57	SE	2	Fine.
6	50	7 0	54	61	29,93	72	SW	1	Foggy.
	60	2 0	60	63	29,93	68	SW	1	Fine.
7	48	7 0	48	60	29,90	71	WSW	1	Foggy.
	66	2 0	66	62	29,95	68	W	1	Fine.
8	51	7 0	54	59	29,90	71	W	1	Foggy.
	66	2 0	71	61	29,96	60	ESE	2	Fine.
9	53	7 0	54	61	29,92	74	NE	1	Rain.
	63	2 0	64	62	29,89	62	E	2	Fine.
10	55	7 0	56	60	29,83	66	N	1	Cloudy.
	61	2 0	58	60	29,84	63	ESE	1	Cloudy.
11	52	7 0	53	60	29,92	66	NNE	1	Cloudy.
	60	2 0	62	61	29,92	58	SSE	1	Fine.
12	47	7 0	49	58	29,85	62	NE	1	Fair.
	57	2 0	66	60	29,85	56	NE	1	Fine.
13	41	7 0	45	57	30,00	61	NNE	2	Fair.
	56	2 0	63	59	30,05	56	NE	2	Fine.
14	41	7 0	46	56	30,23	63	N	1	Cloudy.
	59	2 0	57	57	30,23	56	ESE	2	Fair.
15	45	7 0	47	55	30,19	61	E	2	Fair.
	56	2 0	59	57	30,10	55	E	1	Fine.
16	44	7 0	45	55	29,89	62	E	1	Fair.
	56	2 0	54	56	29,83	62	SE	1	Cloudy.

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1810	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom. Inches.	Hygrometer.	Winds. Points.	Str.	Weather.
	°			°	°		°			
Oct. 17	54	7	0	57	57	29,65	78	NE	1	Fair.
	64	2	0	61	58	29,57	65	SE	1	Cloudy.
18	55	7	0	54	57	29,35	73	W	1	Cloudy.
	64	2	0	63	59	29,46	60	WSW	1	Fair.
19	48	7	0	49	57	29,80	65	N	1	Rain.
	60	2	0	58	58	29,79	62	SW	1	Cloudy.
20	58	7	0	58	58	29,72	66	SW	1	Cloudy.
	65	2	0	60	61	29,66	64	SW	1	Rain.
21	57	7	0	54	56	29,59	70	WSW	1	Rain.
	64	2	0	64	59	29,56	71	SW	2	Rain.
22	58	7	0	58	59	29,30	66	W	2	Cloudy.
	61	2	0	60	59	29,30	56	W	1	Fair.
23	49	7	0	50	57	29,63	61	W	1	Hazy.
	57	2	0	57	61	29,62	56	W	2	Fair.
24	45	7	0	47	58	29,67	62	W	1	Hazy.
	53	2	0	52	60	29,83	59	NNW	1	Fair.
25	42	7	0	42	57	30,14	64	NW	1	Hazy.
	52	2	0	52	57	30,21	60	NNE	1	Fair.
26	40	7	0	41	56	30,35	67	N	1	Cloudy.
	52	2	0	50	57	30,38	57	NE	2	Fair.
27	45	7	0	45	55	30,23	60	NE	1	Hazy.
	49	2	0	49	57	30,14	59	NNE	1	Cloudy.
28	39	7	0	45	55	29,71	69	WSW	1	Rain.
	51	2	0	49	55	29,57	77	N	1	Rain.
29	38	7	0	40	53	29,59	67	N	1	Hazy.
	44	2	0	44	55	29,71	55	N	2	Fine.
30	34	7	0	38	52	29,79	69	N	1	Fair.
	44	2	0	44	53	29,94	62	N	1	Fair.
31	30	7	0	34	51	30,02	64	W	1	Hazy.
	45	2	0	43	51	29,99	64	WSW	1	Cloudy.

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1810	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without.	Therm. within.	Barom.	Hygrometer.		Winds. Points.	Str.	Weather.
	°			°	°	Inches.	°				
Nov. 1	43	7	0	43	51	29,74	73		W	1	Cloudy.
	51	2	0	48	53	29,67	67		NNE	1	Cloudy.
2	40	7	0	41	50	29,73	67		NNE	2	Cloudy. [Much wind last night.
	*	2	0	47	53	29,81	65		NE	2	Cloudy.
3		7	0	43	51	29,95	67		NE	2	Cloudy.
		2	0	45	54	29,97	69		E	2	Cloudy.
4		7	0	42	51	29,87	73		ENE	1	Rain.
		2	0	45	50	29,80	68		N *	1	Cloudy.
5		7	0	36	48	29,67	68		NW	1	Hazy.
		2	0	40	52	29,55	66		W	1	Cloudy.
6		7	0	38	49	29,16	72		W	1	Hazy.
		2	0	42	53	29,10	69		WSW	1	Cloudy.
7		7	0	40	50	28,94	71		E	1	Rain.
		2	0	41	52	28,95	70		E	1	Cloudy.
8		7	0	41	51	29,16	73		E	1	Hazy.
		2	0	46	54	29,05	73		SSE	1	Fair.
9		7	0	38	51	29,44	73		W	1	Hazy.
		2	0	55	53	29,48	68		S	1	Cloudy.
10		7	0	43	51	28,87	77		E	2	Rain.
		2	0	46	54	28,61	79		SW	1	Cloudy.
11		7	0	45	52	29,19	73		W	1	Hazy.
		2	0	45	53	29,34	70		NW	2	Cloudy.
12		7	0	43	51	29,53	72		NE	1	Cloudy.
		2	0	45	53	29,68	65		NNE	1	Cloudy.
13		7	0	37	51	30,04	71		NNE	1	Cloudy.
		2	0	47	53	30,15	64		ENE	1	Fair.
14		7	0	38	51	30,08	63		ENE	2	Cloudy.
		2	0	40	51	29,86	63		E	2	Cloudy.
15		7	0	40	52	29,47	78		W	1	Cloudy.
		2	0	55	55	29,51	62		WSW	1	Cloudy.
16		7	0	50	55	29,24	72		WSW	2	Cloudy.
		2	0	57	58	29,27	67		SW	1	Fair.

* Six's Thermometer blown down and broken.

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1810	Six's Therm. least and greatest Heat.	Time. H.	M.	Therm. without. °	Therm. within. °	Barom. Inches.	Hygrometer. °		Winds. Points.	Str.	Weather.
Nov. 17		7	0	48	56	29,29	68		W	1	Cloudy.
		2	0	56	58	29,33	65		W	1	Fair.
18		7	0	45	56	29,51	68		W	1	Fair.
		2	0	50	57	29,57	61		W	1	Fair.
19		7	0	48	55	29,54	71		S	1	Fair.
		2	0	49	56	29,59	69		WSW	2	Cloudy.
20		7	0	45	56	29,65	69		S	1	Hazy.
		2	0	46	56	29,66	69		NE	2	Cloudy.
21		7	0	50	56	29,41	79		E	1	Rain.
		2	0	55	59	29,39	77		S	2	Cloudy.
22		7	0	48	57	29,51	67		S	2	Cloudy.
		2	0	50	58	29,65	66		S	1	Rain.
23		7	0	47	57	29,82	71		SE	1	Cloudy.
		2	0	50	60	29,44	68		SE	1	Fair.
24		7	0	49	58	29,74	73		ESE	1	Cloudy.
		2	0	50	59	29,67	73		ESE	2	Cloudy.
25		7	0	45	58	29,58	73		SSE	1	Fair.
		2	0	47	58	29,55	72		SSE	1	Rain.
26		7	0	44	55	29,34	73		ESE	2	Cloudy.
		2	0	44	58	29,26	74		E	1	Rain.
27		7	0	43	55	29,19	75		ESE	1	Foggy.
		2	0	46	59	29,05	73		E	1	Cloudy.
28		7	0	43	55	29,02	74		W	1	Cloudy.
		2	0	47	59	28,99	67		SW	1	Fair.
29		7	0	40	55	28,96	71		W	1	Cloudy.
		2	0	42	55	28,96	75		SSW	2	Cloudy.
30		7	0	37	54	29,13	72		W	1	Cloudy.
		2	0	42	53	29,16	74		ESE	2	Rain.

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1810	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.		Winds.		Weather.
		H.	M.			Inches.			Points.	Str.	
	°			°	°		°				
Dec. 1		8	0	37	53	29,50	71		W	1	Foggy.
		2	0	42	56	29,58	68		SW	1	Fair.
2		8	0	35	51	29,95	67		N	1	Fair.
		2	0	40	53	29,93	68		N	1	Fine.
3		8	0	33	49	29,94	70		SE	1	Fair.
		2	0	40	50	29,88	72		E	1	Rain.
4		8	0	40	49	29,91	78		SW	1	Foggy.
		2	0	47	51	29,93	76		WSW	1	Fair.
5		8	0	45	50	29,92	78		W	1	Fair.
		2	0	49	54	29,92	74		W	1	Cloudy.
6		8	0	47	53	29,59	63		SSW	2	Rain.
		2	0	49	54	29,56	75		S	1	Cloudy.
7		8	0	45	53	29,48	73		W	1	Cloudy.
		2	0	47	54	29,31	68		W	1	Cloudy.
8		8	0	39	53	29,47	70		W	1	Cloudy.
		2	0	40	54	29,60	71		WSW	1	Rain.
9		8	0	33	50	29,83	68		W	1	Cloudy.
		2	0	37	52	29,86	66		W	1	Cloudy.
10		8	0	37	49	29,61	71		E	1	Rain.
		2	0	47	50	29,20	75		SW	2	Rain.
11		8	0	36	48	29,50	74		NNW	1	Cloudy.
		2	0	39	51	29,73	66		N	2	Fair.
12		8	0	35	48	29,75	70		E	1	Cloudy.
		2	0	49	51	29,29	76		SW	1	Rain.
13		8	0	43	48	29,74	74		W	1	Cloudy.
		2	0	47	54	29,94	67		S	1	Cloudy.
14		8	0	49	53	29,84	75		W	1	Cloudy.
		2	0	47	54	29,63	60		W	1	Cloudy.
15		8	0	42	52	29,83	69		W	1	Cloudy.
		2	0	46	53	29,89	67		WNW	1	Fair.
16		8	0	41	51	30,18	69		N	1	Cloudy.
		2	0	41	51	30,27	70		W	1	Foggy.

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for December, 1810.

1810	Six's Therm. least and greatest Heat.	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Winds.		Weather.
		H.	M.			Inches.		Points.	Str.	
	°			°	°		°			
Dec. 17		8	0	45	52	30,14	73	W	1	Fogg.
		2	0	46	52	29,68	69	W	1	Cloudy.
18		8	0	47	52	29,63	77	S	1	Rain.
		2	0	47	52	29,58	70	WSW	1	Rain.
19		8	0	42	50	29,46	68	W	1	Cloudy.
		2	0	42	52	29,40	68	W	1	Rain.
20		8	0	36	49	29,35	69	W	1	Fair.
		2	0	46	53	29,46	70	W	1	Fair.
21		8	0	42	51	29,32	69	W	1	Rain.
		2	0	41	51	29,28	65	W	1	Fair.
22		8	0	41	50	29,62	65	W	1	Rain.
		2	0	46	52	29,66	70	SW	1	Rain.
23		8	0	47	52	29,38	69	W	2	Rain.
		2	0	48	52	29,43	66	W	1	Fair.
24		8	0	43	50	29,26	73	N	1	Rain.
		2	0	43	50	29,46	71	W	1	Fair.
25		8	0	47	50	28,98	72	SW	2	Rain.*
		4	0	45	50	29,03	66	W	1	Cloudy.
26		8	0	43	48	29,51	66	W	1	Fair.
		4	0	47	52	29,77	62	W	1	Cloudy.
27		8	0	47	51	29,39	72	SW	2	Rain.
		4	0	45	52	29,58	61	W	1	Fair.
28		8	0	39	50	30,04	62	NNW	1	Cloudy.
		4	0	39	51	30,16	61	NNW	1	Fair.
29		8	0	33	48	30,35	61	NNW	1	Fair.
		4	0	37	50	30,60	63	NW	1	Fair.
30		8	0	32	47	30,50	64	N	1	Fair.
		4	0	37	47	30,45	67	N	1	Fair.
31		8	0	31	45	30,49	65	NE	1	Fair.
		4	0	32	48	30,44	64	N	1	Fair.

* A violent gale of wind in the night.

1810.	Six's Therm. without.			Thermometer without.			Thermometer within.			Barometer.*			Hygrometer.			Rain.
	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least height.	Mean height.	
	Deg.	Deg.	Deg.	Deg	Deg.	Deg.	Deg.	Deg.	Deg.	Inches.	Inches.	Inches.	Deg.	Deg.	Deg.	Inches.
January	51	14	36,3	51	18	36,6	57	42	49,2	30,44	29,73	30,12	75	60	70,7	
February	54	18	40,6	54	19	40,8	56	43	50,8	30,44	29,01	29,86	78	57	69,1	
March	59	28	44,1	58	30	44,6	61	50	54,9	30,05	28,84	29,69	76	49	64,4	
April	69	31	48,7	69	35	49,9	62	50	55,1	30,21	29,29	29,80	76	46	62,2	
May	67	34	52,3	67	40	53,7	61	53	57,5	30,38	29,35	29,87	75	48	58,1	
June	78	45	61,5	78	52	62,0	66	59	62,0	30,38	29,68	30,07	64	48	56,7	
July	78	51	63,1	77	56	64,7	68	62	64,2	30,16	29,31	29,79	66	50	58,8	
August	80	50	63,3	80	51	64,3	71	59	63,9	30,28	29,45	29,73	72	51	61,3	
September				72	51	62,1	76	60	63,9	30,36	29,64	30,02	79	51	63,2	
October				71	34	54,1	65	51	56,9	30.38	29,30	29,91	78	55	63,9	
November				57	36	45,1	60	48	52,6	30,15	28,61	29,45	79	63	70,2	
December				49	31	40,5	56	45	51,0	30,60	28,98	29,72	78	61	69,1	
Whole year						51,5			56,8			29,84			64,0	

* The quicksilver in the bason of the barometer, is 81 feet above the level of low water spring tides at Somerset-house.